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THE MEASUREMENT OF THE STATIC SWITCHING PROPERTIES OF JUNTION TRANSISTORS

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by

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Submitted in partial fulfillment
of the requirements
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The transistor, first developed by J. Bardeen and W. H. Brattain of Bell Telephone Laboratories in 1948, grew out of a series of deliberate investigations of semiconductor properties. Interest in semiconductors is recorded as early as 1833 when Faraday found that the electrical resistance of silver sulfide decreased with increasing temperature while the resistance of metals increased with temperature. Observation of the photoelectric effect by Becquerel followed in 1839 and in 1874 it was found by F. Braun and A. Schuster that the electrical resistance of contacts between certain materials did not appear to obey Ohm's law but, instead were non-linear functions of the polarity and magnitude of the applied voltage. Progress in semiconductor research was for many years very slow; in fact, it was not until the early 1940's after the pressure of war had been released that the exploitation of the semiconductor field was begun in earnest. Silicon and germanium diodes had been developed during the war in answer to the need for detectors at radar frequencies but their mechanism of rectification was poorly understood. Accordingly, a group of scientists at Bell Telephone Laboratories was organized for the purpose of developing the theory of conduction in semiconductors. It was with this group in 1948, during an investigation of semiconductor surface states, that J. Bardeen and W. H. Brattain developed the point-contact transistor. This development was heralded as a milestone in semiconductor research. Soon thereafter, a short step was made from the point-contact transistor to the junction transistor. (10)

Literature describing the development and uses of junction transistors has multiplied in recent years. Among many uses, interest has fallen in particular, on the use of a junction transistor as a switch. This paper treats of this use of a junction transistor and, more particularly, the measurement of the static characteristic of a transistor when so used. Chapter I explains, qualitatively, what happens within a transistor when it is used as a switch; then, equations are developed to show the switching action in quantitative terms. Chapter II considers various aspects of practical switches and then discusses the advantages and limitations of junction transistors in switching applications. Chapter III follows with a development of the circuitry for a quick check device to measure the static switching parameters of a transistor. Chapter IV concludes the paper by comparing actual measurements with the corresponding theoretical predictions based on equations in Chapter I.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS AND ABBREVIATIONS

a_{11}, a_{12}, a_{21}, a_{22}	Coefficients of the terms in equations (1-3) and (1-4). These coefficients are determined by the geometry of the transistor and the electrical properties of the semiconducting material used in the transistor.
a-c	Either "alternating current" or "alternating."
d-c	Either "direct current" or "direct."
D_n	The diffusion constant for electrons in the emitter of a p-n-p transistor.
D_n'	The diffusion constant for electrons in the collector of a p-n-p transistor.
D_p	The diffusion constant for holes in the base of a p-n-p transistor.
e	The natural logarithm base.
E_o	Output voltage.
I	Current.
I_b	Base current; positive sense into the transistor.
I_c	Collector current; positive sense into the transistor.
I_{c_n}	The component of collector current contributed by electron flow.
I_{c_p}	The component of collector current contributed by hole flow.
I_{co}	The collector junction voltage saturation current which flows when $I_e = 0$.
I_e	Emitter current; positive sense into the transistor.
I_{e_n}	The component of emitter current contributed by electron flow.
I_{e_p}	The component of emitter current contributed by hole flow.
I_{eo}	The emitter junction voltage saturation current which flows when $I_c = 0$.
$I_p(x)$	That part of the current flow in the x-direction at any point, x, in the base due to the movement of holes.
I_{R_f}	The current flowing through resistor R_f.

I_s	The reverse saturation current across a p-n junction due to the flow of minority carriers.
j	$\sqrt{-1}$
k	Boltzmann's constant = 1.38×10^{-23} Joules/^{o}K molecule.
L_n	The diffusion length for electrons in the emitter of a p-n-p transistor; the distance from the base into the emitter at which electron concentration has decreased by a factor 1/e.
L_n'	The diffusion length for electrons in the collector of a p-n-p transistor; the distance at which electron concentration has decreased by a factor of 1/e.
L_p	The diffusion length for holes in the base of a p-n-p transistor; the distance from the emitter into the base at which the hole concentration has decreased by a factor of 1/e.
m	The ratio of the actual I_c to the I_c which would flow if ionization were not present at the collector junction.
n	Electron concentration.
n_p	The thermal equilibrium concentration of electrons in p-type semiconducting material.
p	Hole concentration.
p_n	The thermal equilibrium concentration of holes in n-type semiconducting material.
$P_o(x)$	The static component of hole concentration at any point, x, in the base of the p-n-p transistor discussed in Appendix I.
$p_1(x)$	The time varying component of hole concentration at any point, x, in the base of the p-n-p transistor discussed in Appendix I.
p-n	An adjective used to describe the junction between a p-type semiconductor and a n-type semiconductor.
q	The electrical charge of an electron = 1.6×10^{-19} coulombs.
Q_p	As used in Appendix I it is the total charge in a volume of the transistor base of unit cross-section and length, W, where the unit cross-section is taken in a plane parallel to the base surface and the length, W, the base thickness, is taken along the x-direction.

r_{ce}	The low frequency dynamic collector resistance, analogous to plate resistance in a vacuum tube.
R_{ce}	The d-c collector resistance, analogous to the beam resistance of a vacuum tube.
R_{cs}	The series resistance of the collector region excluding the collector junction.
r_{ec}	The low frequency dynamic emitter resistance of a transistor operated in its inverted configuration. It is analogous to r_{ce} of a transistor operated in its normal configuration.
R_{ec}	The d-c emitter resistance of a transistor operated in its inverted configuration. It is analogous to R_{ce} of a transistor operated in its normal configuration.
R_{es}	The series resistance of the emitter region excluding the emitter junction.
R_f	The feedback resistor shown in figure (7).
R_i	The input resistor shown in figure (7).
R_1, R_2, etc.	The base current ladder resistors shown in figure (11).
t	Time.
T	Temperature in degrees Kelvin.
T_1, T_2, etc.	Transistors, as shown in figures (3) and (5).
V_{be}	The base voltage of a transistor measured with respect to its emitter.
V_{cb}	The collector voltage of a transistor measured with respect to its base.
V_{ce}	The collector voltage of a transistor measured with respect to its emitter.
V_{eb}	The emitter voltage of a transistor measured with respect to its base.
V_{ec}	The emitter voltage of a transistor measured with respect to its collector.
V_1, V_2, etc.	Vacuum tubes such as are shown in figure (7).

W	The thickness of the base of the transistor discussed in Appendix I.
x	Distance along the axis of the transistor described in Appendix I measured from the emitter junction with positive sense into the base.
α_i	The inverted d-c current gain of the transistor; the fraction of the collector current that reaches the emitter, defined by the equation, $\alpha_i = \left.\frac{I_e}{I_c}\right\|_{V_{eb}=0}$.
α_n	The normal d-c current gain of the transistor; the fraction of the emitter current that reaches the collector, defined by the equation, $\alpha_n = \left.\frac{I_c}{I_e}\right\|_{V_{cb}=0}$
α normal	Same as α_n.
β	Base current multiplication factor = $\frac{\alpha}{1-\alpha}$
τ_p	The "hole lifetime." It is the time for a group of holes injected into a piece of n-type semiconducting material to be reduced in number by a factor of 1/e.
φ	The voltage of the p-side of a p-n junction measured with respect to the n-side of the junction.
φ_c	φ at the collector junction.
φ_e	φ at the emitter junction.
ω	Angular frequency in radians per unit time

CHAPTER I

A DISCUSSION OF THE LARGE SIGNAL OPERATION OF JUNCTION TRANSISTORS

The aim of this chapter is to explain briefly and in qualitative terms what happens within a transistor when it is used as a switch. The equations which govern the transistor switching action will then be presented and some of the limitations inherent in these equations will be discussed. Attention will be confined to just the d-c or static behavior of the transistor. The a-c effects, although of great importance in the operation of a transistor as a switch, will not be considered in this paper.

In presenting a picture of the operation of a junction transistor, it is first desirable to describe the p-n junction itself. A p-n junction is a boundary between two slightly different types of semiconducting materials. On the n-type side of the junction the mobile charge carriers are mostly electrons drifting within a fixed structure of bound charges. These bound charges are carried on penta-valent impurity atoms which are imperfections in a crystal lattice consisting of tetra-valent semiconductor atoms. In the n-type crystal, these mobile electrons are called the "majority carriers." A few positive carriers are also present in the n-material and these are called the "minority carriers." On the p-type side of the junction the mobile charge carriers are mostly positive "holes" drifting within a fixed structure of bound charges. These bound charges are held on tri-valent impurity atoms which are imperfections in a crystal lattice of tetra-valent semiconductor atoms. In the p-material the hole is the "majority

carrier" and the electron, the "minority carrier." The "hole" does not really exist as a discrete particle; it is just a convenient abstraction used to explain the net behavior of electrons in a semiconducting crystal lattice containing imperfections which produce an electron deficiency. An adequate description of the hole is very involved but a hole behaves as if it were a positive electron with the same mass as an ordinary electron but with slightly less mobility than an electron (13).

In the region near a p-n junction, electrons in the n-material are repelled from the junction by the bound negative charge on the p-side of the junction. Similarly, holes in the p-material are repelled from the junction by the bound positive charge on the n-side of the junction. There is thus created a potential hill due to the bound charge at the junction which tends to oppose the migration of majority carriers across the junction. (Note that once a majority carrier crosses a p-n junction it becomes by definition a minority carrier.) A voltage, φ, external to the p-n junction will now be defined as a voltage applied to the p-material measured with respect to the n-material. When φ is applied to a p-n junction and is made more positive than a few tenths of a volt, the potential hill at the junction is overcome and copious flow of majority carriers results. This condition will be referred to as "current saturation." When φ is made a few tenths of a volt negative, the junction current becomes quite small and is due to the flow of minority carriers across the junction. Since these minority carriers are relatively few and their rate of generation is substantially independent of the junction voltage, this current remains fairly constant even when φ is made several volts negative. This condition will be called "voltage

saturation." This non-ohmic behavior of a p-n junction is described fairly well by the equation, (8),

$$I = I_s \left[e^{q\varphi/kT} - 1 \right] \qquad (1\text{-}1)$$

where, I = the junction current.

I_s = the reverse saturation current across the junction due to the flow of minority carriers.

q = electronic charge = 1.6×10^{-19} coulombs.

φ = voltage of the p-material with respect to the n-material.

k = Boltzmann's constant = 1.38×10^{-23} Joules/°K molecule.

T = temperature in degrees Kelvin.

A junction transistor may be thought of as two p-n junctions placed parallel to each other, and separated by just a very thin layer of semi-conducting material called the "base." When the transistor is used as an amplifier, one of these junctions is forward-biased so that current flows across it easily; this junction is called the "emitter junction." It is usually designed to operate so that most of the junction current consists of carriers flowing into the base. The other junction is called the "collector junction." It is reverse-biased so that in the absence of the emitter junction, just the voltage saturation current would flow across it. The emitter, however, injects a large number of minority carriers into the base layer. These carriers do not immediately recombine with the majority carriers already in the base, but instead they diffuse across the base layer to the collector junction where most of them are swept by the reverse bias potential across that junction into the collector region, thus increasing the collector current I_c considerably above its normal voltage saturation value, I_{co}. If it is assumed that there is no voltage

drop within the transistor, except across the junctions, minority carriers which have been injected by the emitter into the base will proceed across the base only by means of diffusion. If φ_c, the voltage of the p-side of the collector junction measured with respect to the n-side, is more negative than a few tenths of a volt, the current across the collector junction, I_c, will be almost independent of it. I_c will be determined, instead, by the geometry of the transistor, the parameters of the diffusion process in the base layer and the rate at which minority carriers are emitted into the base layer. I_c can be controlled because the rate at which minority carriers are emitted into the base can be varied at will.

The discussion up to this point has presented a brief qualitative picture of how a transistor operates. There is no reason why the actions which have been described so far cannot be put into analytical form and, in fact, this has already been done by Ebers and Moll (5). The following development of the equations which describe the transistor switch will very closely parallel the development contained in the Ebers and Moll paper (5) and in Chapter Eight of TRANSISTOR ELECTRONICS (7).

For a transistor to be used as a switch, it is necessary that it be driven between certain conditions of saturation. The condition for the "on" state is that the emitter junction be forward biased so heavily that a small change in base current, I_b, will have very little effect on collector current, I_c, or in other words, that the transistor be current saturated. The condition for the "off" state is that both junctions be reverse biased so that a small change in the voltage from base to emitter, V_{be}, will have very little effect on I_c or simply that the transistor be

voltage saturated. These two conditions of drive are the extremes between which an amplifier would operate. The switch, however, is ideally in either one or the other of these saturated states all the while, except for a brief period of transition between states. Therefore, one can define two regions of saturated operation separated by the usual region for operation of an amplifier. Ebers and Moll (5) have defined these regions as follows:

Region I, the "off" or voltage saturation region. Its boundaries are defined by the conditions that both the emitter and collector junctions conduct reverse bias currents.

Region II, the normal operation region for a transistor amplifier. Here, the emitter junction carries forward bias current, while the collector junction carries reverse bias current.

Region III, the "on" or current saturation region. Its boundaries are such that the emitter and collector both carry forward bias current.

It is to be noted that region II also includes the "inverted transistor" condition where the electrode designated "collector" is used as emitter and the electrode designated "emitter" is used as collector. When it is desired to distinguish between these two conditions, the former will be referred to as "normal region II" and the latter as "inverted region II." In general, the words "normal" and "inverted," when applied to a transistor configuration, or the letters "n" and "i," when used as subscripts, will refer to these uses of the emitter and collector electrodes. An exception to this rule will be made for the subscript "n" which is used for another purpose in Appendix I and in equations (1-2), (1-3), (1-4) and (1-18) of this chapter.

The relationships between the emitter and collector voltages and the emitter and collector currents of a transistor may be found by, first, setting up a geometrical model of the transistor. Then, by considering the diffusion of carriers through the homogeneous regions of the transistor and, finally, by accounting for the behavior of voltages and carrier flow at the junctions; this establishes the boundary conditions for the diffusion process in the homogeneous regions of the transistor.

The result of this process is a set of equations which describes the terminal voltages and currents of the transistor. There is no necessary reason to place restrictions on the transistor geometry in carrying out this process and, in fact, Ebers and Moll (5) in their paper have shown a solution for a very general geometrical arrangement. The details of the process are made clearer, however, if the transistor geometry is arranged so that variations in only one dimension have to be considered; furthermore, notation is simplified if the derivation is limited to either a p-n-p or an n-p-n transistor. Appendix I shows in detail the derivation of the diffusion equation, its solution and the application of boundary conditions to arrive at the equations describing the terminal voltages and currents of a p-n-p transistor of simplified geometrical arrangement.

The differential equation which describes the diffusion of holes through a slab of homogeneous semiconductor, such as germanium, containing an n-type impurity is,

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} + \frac{p_n - p}{\tau_p} \qquad (1\text{-}2)$$

where,

p = the concentration of holes at any time, t, and at any point, x, in the slab.

t = time.

x = distance into the slab measured along the normal to its surface.

p_n = the thermal equilibrium concentration of holes in the n-type germanium.

τ_p = the "hole lifetime." It is the time required for a group of holes injected into the slab to be reduced by a factor of $1/e$. (e is the natural log base.)

D_p = the diffusion constant for holes in the slab of n-type germanium. It has the dimensions $(\text{length})^2/\text{time}$.

Let one surface of this slab form the emitter p-n junction and let the other surface parallel to the first and separated from it by a distance W, form the collector p-n junction. Equation (1-2) is then solved for the d-c case. The hole concentration at each junction (see Appendix I, equations A1-14 and A1-15) is applied as the boundary condition to this solution to obtain an equation for the hole concentration (equation A1-19) at any point within the base. The hole current is found from the gradient of the hole concentration equation. Particular solutions of this gradient equation (equation A1-21) at the emitter and collector junctions give the hole currents at these junctions (equations A1-22 and A1-23). The electron currents at these junctions are found by analogy from the corresponding hole current equations. (Equations A1-24 and A1-25). Electron and hole currents at each junction are then added to obtain the total current at each junction. When this is done, the result is,

$$I_e = a_{11}\left(e^{\frac{q\varphi_c}{kT}} - 1\right) + a_{12}\left(e^{\frac{q\varphi_e}{kT}} - 1\right) \tag{1-3}$$

and

$$I_c = a_{21}\left(e^{\frac{q\varphi_c}{kT}} - 1\right) + a_{22}\left(e^{\frac{q\varphi_e}{kT}} - 1\right) \tag{1-4}$$

where,

$$a_{11} = \frac{q D_p p_n}{L_p}\left[\coth\frac{W}{L_p} + \frac{D_n n_p L_p}{D_p p_n L_n}\right]$$

$$a_{12} = a_{21} = \frac{-q D_p p_n}{L_p} \operatorname{csch}\frac{W}{L_p}$$

$$a_{22} = \frac{q D_p p_n}{L_p}\left[\coth\frac{W}{L_p} + \frac{D_n n_p' L_p}{D_p p_n L_n'}\right]$$

and where, q = electronic charge = 1.6×10^{-19} coulombs.

k = Boltzmann's constant = 1.38×10^{-23} Joules/°K molecule.

T = Temperature in degrees Kelvin.

φ_e = voltage of the p-side of the emitter measured with respect to the n-side.

φ_c = voltage of the p-side of the collector junction measured with respect to the n-side.

D_p = Diffusion constant for holes in the base (n-material), Length^2/ time.

D_n = Diffusion constant for electrons in the emitter (p-material), Length^2/ time.

D_n' = Diffusion constant for electrons in the collector (p-material), Length^2/ time.

L_p = Diffusion length for holes in the base; distance into the base at which the hole concentration has decreased by a factor of 1/e.

L_n = Diffusion length for electrons in the emitter; distance from the base into the emitter at which electron concentration has decreased by a factor of 1/e.

L_n' = Diffusion length for electrons in the collector; distance from the base into the collector at which electron concentration has decreased by a factor of 1/e.

p_n = the thermal equilibrium concentration of holes in n-type material.

n_p = the thermal equilibrium concentration of electrons in p-type material.

W = the width of the base layer.

I_e = emitter current measured with positive sense into the transistor.

I_c = collector current measured with positive sense into the transistor.

The form of these coefficients makes them of value in visualizing the various factors that contribute to the d-c behavior of a transistor. They might possibly be of interest in the design of a transistor, but they are not well adapted to predicting the large signal behavior of a transistor from known or measurable small signal parameters. Furthermore, the lack of generality of the model for which they were derived also limits their usefulness.

Ebers and Moll (5) have derived expressions for a_{11}, a_{12}, a_{21} and a_{22} of equations (1-3) and (1-4) in terms of small signal parameters. Their equations are quite general and may be applied to any n-p-n or p-n-p transistor. These coefficients which are derived in Appendix II are:

$$a_{11} = \frac{-I_{eo}}{1-\alpha_n \alpha_i} \qquad (1\text{-}5)$$

$$a_{12} = \frac{\alpha_i I_{co}}{1-\alpha_n \alpha_i} \qquad (1\text{-}6)$$

$$a_{21} = \frac{\alpha_n I_{eo}}{1-\alpha_n \alpha_i} \qquad (1\text{-}7)$$

$$a_{22} = \frac{-I_{co}}{1-\alpha_n \alpha_i} \qquad (1\text{-}8)$$

where, I_{eo} = the emitter junction voltage saturation current which flows when collector current, $I_c = 0$.

I_{co} = the collector junction voltage saturation current which flows when emitter current, $I_e = 0$.

α_n = the normal d-c current gain of the transistor; the fraction of the emitter current that reaches the collector, defined by the equation, $\alpha_n = \left.\frac{I_c}{I_e}\right|_{V_{cb}=0}$

α_i = the inverted d-c current gain of the transistor; the fraction of the collector current that reaches the emitter, defined by the equation, $\alpha_i = \left.\frac{I_e}{I_c}\right|_{V_{eb}=0}$

V_{cb} = the voltage applied to the collector measured with respect to the base region.

V_{eb} = the voltage applied to the emitter measured with respect to the base region.

It was shown in Appendix I that $a_{12} = a_{21}$ and this is true in general (5); therefore, it follows from equations (1-6) and (1-7) that,

$$\alpha_n I_{eo} = \alpha_i I_{co} \qquad (1\text{-}9)$$

Equations (1-3) and (1-4) may now be re-written in terms of these "a" coefficients:

$$I_e = \frac{1}{1-\alpha_n\alpha_i}\left[-I_{eo}\left(e^{\frac{q\varphi_e}{kT}}-1\right) + \alpha_i I_{co}\left(e^{\frac{q\varphi_c}{kT}}-1\right)\right] \qquad (1\text{-}10)$$

$$I_c = \frac{1}{1-\alpha_n\alpha_i}\left[\alpha_n I_{eo}\left(e^{\frac{q\varphi_e}{kT}}-1\right) - I_{co}\left(e^{\frac{q\varphi_c}{kT}}-1\right)\right] \qquad (1\text{-}11)$$

These equations show very clearly the d-c behavior of a junction transistor with respect to its terminal voltages and currents. Equation (1-11), for example, shows that I_c is dependent only upon φ_e when φ_c is more than a few tenths of a volt negative*. It shows, also, that in region I, $I_c < I_{co}$ and that φ_e, φ_c, and I_c are relatively independent of each other. In region III it shows that changes in I_c produce little change in φ_e and φ_c. These two equations, then, within the limits of the assumptions upon which they were derived, would describe very well the behavior of a transistor

*The quantity kT/q = .026 volts at room temperature, 300°K or 80°F.

in a switching application.

Equations for V_{ce} and V_{ec} will be found to be useful in Chapter IV and will therefore be presented here. These equations follow from (1-10) and (1-11) and are derived in Appendix III:

$$V_{ce} = \pm \frac{kT}{q} \ln \left\{ \frac{\alpha_i \left[1 - \frac{I_c}{I_b} \frac{(1-\alpha_m)}{\alpha_m}\right]}{1 + \frac{I_c}{I_b}(1-\alpha_i)} \right\} \tag{1-12}$$

$$V_{ec} = \pm \frac{kT}{q} \ln \left\{ \frac{\alpha_m \left[1 - \frac{I_e}{I_b} \frac{(1-\alpha_i)}{\alpha_i}\right]}{1 + \frac{I_e}{I_b}(1-\alpha_m)} \right\} \tag{1-13}$$

where, V_{ce} = collector voltage measured with respect to the emitter.

V_{ec} = emitter voltage measured with respect to the collector.

I_c = collector current; positive sense is into the collector.

I_e = emitter current; positive sense is into the emitter.

I_b = base current; positive sense is into the base.

The (+) sign is to be chosen for p-n-p transistors and the (-) sign, for n-p-n transistors. Other d-c parameters which are of interest in the design of switching circuits are,

$$r_{ce} \equiv \frac{dV_{ce}}{dI_c} = \pm \frac{kT}{q} \left\{ -\frac{1-\alpha_i}{I_b + I_c(1-\alpha_i) + \frac{\alpha_i}{\alpha_m} I_{co}} - \frac{1-\alpha_m}{a_m\left(I_b - I_c\frac{1-\alpha_m}{\alpha_m}\right) - I_{co}} \right\} \tag{1-14}$$

r_{ce} is the low frequency dynamic collector resistance, analogous to r_p for a vacuum tube. r_{ec} is the inverted case of r_{ce}.

$$r_{ec} \equiv \frac{dV_{ce}}{dI_c} = \pm \frac{kT}{q} \left\{ -\frac{1-\alpha_m}{I_b + I_e(1-\alpha_m) + \frac{\alpha_m}{\alpha_i} I_{eo}} - \frac{1-\alpha_i}{\alpha_i\left(I_b - I_c\frac{1-\alpha_i}{\alpha_i}\right) - I_{eo}} \right\} \tag{1-15}$$

R_{ce} is the d-c collector resistance, analogous to the beam resistance of a vacuum tube and R_{ec} is the inverted case of R_{ce} in the following equations:

$$R_{ce} = \frac{V_{ce}}{I_c} = \pm \frac{kT}{qI_c} \ln \left\{ \frac{\alpha_i \left[1 - \frac{I_c}{I_b} \frac{(1-\alpha_n)}{\alpha_n} \right]}{1 + \frac{I_c}{I_b}(1-\alpha_i)} \right\} \tag{1-16}$$

$$R_{ec} = \frac{V_{ec}}{I_e} = \pm \frac{kT}{qI_e} \ln \left\{ \frac{\alpha_n \left[1 - \frac{I_e}{I_b} \frac{(1-\alpha_i)}{\alpha_i} \right]}{1 + \frac{I_e}{I_b}(1-\alpha_n)} \right\} \tag{1-17}$$

Note again that the (+) sign is to be used for p-n-p transistors and the (-) sign, for n-p-n transistors. Derivations of equations (1-14) and (1-15) may be found in Appendix IV.

It is interesting to note that the inverted equations in each case so far presented may be obtained from the normal equations if one merely interchanges the subscripts "c" and "e," and "n" and "i." That this should be true is not surprising because there is no difference in the nature of the actions occurring at collector and emitter, except as is determined by the polarity and magnitude of bias currents at these junctions.

These equations describe large signal behavior of a transistor fairly well but the assumptions, as stated in Appendix I, upon which their derivation is based limit their range of application. Consideration will be given here to some of these limitations.

Equation (1-11), $I_c = \frac{\alpha_n I_{eo}}{(1-\alpha_n \alpha_i)}\left(e^{\frac{q\varphi_e}{kT}} - 1\right) - \frac{I_{co}}{1-\alpha_n\alpha_i}\left(e^{\frac{q\varphi_c}{kT}} - 1\right)$,

predicts that as φ_c is made increasingly negative, other variables being held constant, the change in I_c will become negligibly small. There is

no indication that this condition will not hold even at extremely large negative voltages. Actually, however, as the inverse voltage is increased across a p-n diode junction, a critical voltage, the Zener voltage, is reached above which the current through the junction increases very rapidly. The Zener breakdown of a p-n junction is analogous to field emission in metals. It represents the electric field strength which is needed to tear otherwise stable electrons from their valence linkages (7). Zener breakdown is usually very sharp and when it has occurred the voltage across the diode remains constant over a rather wide range of current. In a transistor, other voltage effects usually become prominent before the Zener voltage is reached. One of these is avalanche breakdown, which at one time was not distinguished from Zener breakdown, (12). It occurs in collector-base junctions when the space-charge region (i.e. the region across the junction where an electric field exists and carrier transport is not primarily a result of the diffusion process) is fairly wide. When reverse bias on the collector is increased, the space charge region widens, extending mostly into the base, since the base region is usually made of higher resistivity than the collector region. The carriers are accelerated toward the collector and as collector voltage is increased, an increasingly larger number of these carriers have enough kinetic energy to rupture the valence linkages between atoms in the crystal lattice. In this way a larger number of electron-hole pairs is formed and the current across the junction is increased, (12).

In some transistors, the collector space-charge region can be widened until it touches the emitter junction before enough ionization has occurred to cause avalanche. This condition is called "punch-through."

When the base has been punched through, there is no effective base region left and the situation is as if the emitter had been shorted through to the collector. Of course, no transistor action can be obtained in this condition (12).

Associated with this narrowing of the effective base layer, is an increase in α. That this increase should be expected can be seen from the equation for α derived on the basis of diffusion effects.*

$$\alpha_{normal} = \frac{1}{\cosh\frac{W}{L_p} + \frac{D_n n_p L_p}{D_p p_n L_n}\sinh\frac{W}{L_p}} \tag{1-18}$$

The ionization process which, if it is allowed to become prominent, leads to avalanche, also contributes to an effective increase in α. A multiplication ratio, m, can be defined as the ratio of the actual current across the junction to the current which would flow in the absence of this ionization effect. This factor, m, has a value close to unity for low collector voltages and it increases rapidly as the avalanche voltage, V_a, is approached. Most good junction transistors have normal α's in the range of values between 0.9 and 1.0; therefore, only a small increase in m is needed to make the product, $\alpha m = 1$. Interesting effects occur in the region where m begins to become appreciably larger than unity. The formula for base current multiplication factor, β,

$$\beta = \frac{\alpha}{1-\alpha} \tag{1-19}$$

must be modified to,

$$\beta = \frac{\alpha m}{1-\alpha m} \tag{1-20}$$

*See Appendix V.

Variation of β and αm with respect to collector voltage

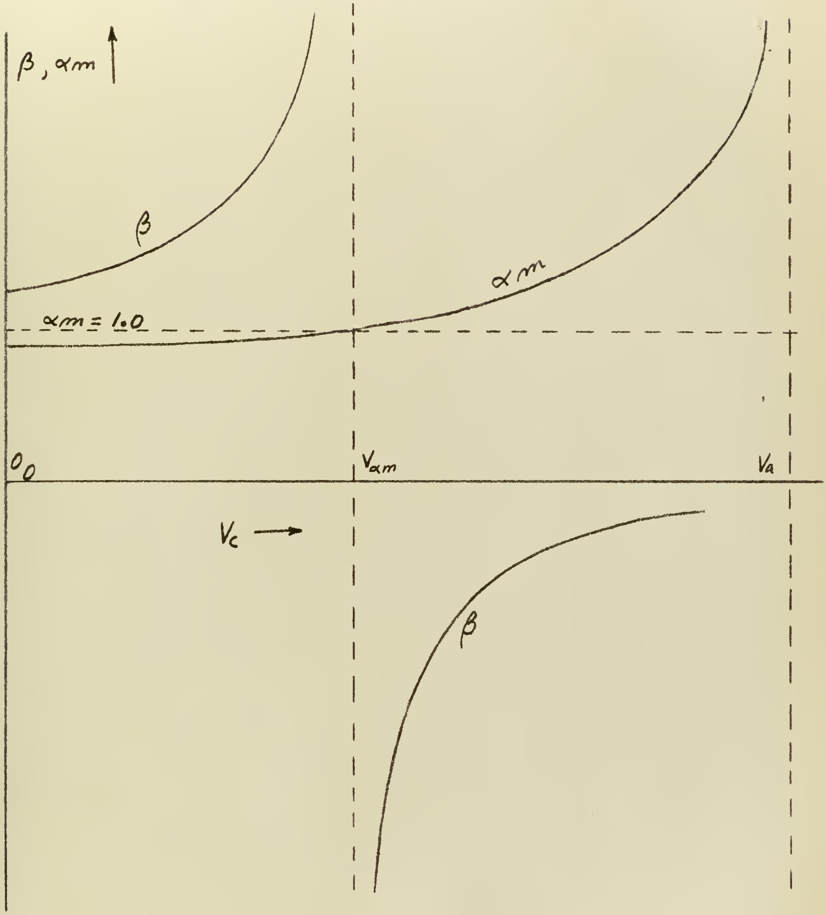


Figure (1)

and when $\alpha m = 1$, β becomes infinite; as αm is made larger than unity, β becomes negative (12). Figure (1) shows α_m and β plotted against collector voltage. A voltage, $V_{\alpha m}$, has been defined as the voltage at which $\alpha m = 1$,(12). It is possible to operate a transistor in the range between $V_{\alpha m}$ and V_a where $\alpha > 1$ and $\beta < 0$. Operation in this region opens the possibility of obtaining the characteristics of a point-contact transistor in a junction unit, (11). Because operation above $V_{\alpha m}$ is associated with high collector currents, it is accompanied by an increase in temperature at the collector junction. An increase in junction temperature causes a decrease in the factor, $(e^{\frac{q\phi_c}{kT}} -1)$* and, therefore, an increase in I_o. This cumulative process, which has been termed "collector runaway," can result in destruction of the transistor, (12).

None of these effects just described is predicted by the equations which have been developed from a consideration of the carrier diffusion equation. Whether any of them can be predicted from small signal parameters alone seems very unlikely. One is led, then, to the need for more information than can be obtained from small signal parameters.

*See equation (1-11).

CHAPTER II

A DISCUSSION OF SWITCHES AND AN EVALUATION OF THE JUNCTION TRANSISTOR SWITCH

One might specify the properties of an ideal switch as,

1. An infinite admittance in the "on" state.

2. An infinite impedance in the "off" state.

3. An ability to shift instantaneously between "off" and "on" states.

4. The consumption of as small an amount of controlling power as might be desired.

Such a device, of course, does not exist. Practical switches are limited in many respects which depend, largely, on the design of the switch. These are some of the shortcomings which limit the usefulness of practical switches:

1. A voltage drop exists when a current flows through it.

2. The amount of current that can be conducted through the switch is limited by the power which can be dissipated in the switch.

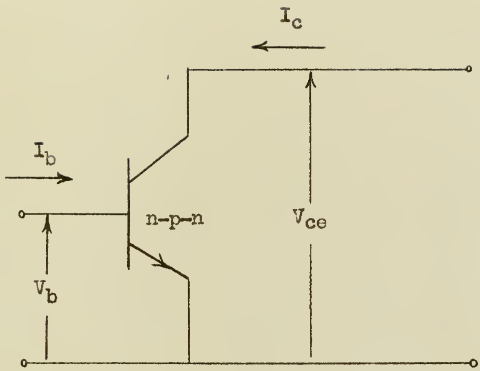
3. No practical switch can be shifted instantaneously between "on" and "off" states; therefore, the maximum energy which can be dissipated by the switch during this transition may be a limitation of the switch.

4. Transition between "on" and "off" states is frequently accompanied by a large amount of noise; moreover, the instants of "make" and "break" are often poorly defined.

5. A switch in its "off" state will conduct when,

   a. Dielectric breakdown occurs.

   b. A voltage is applied and a leakage resistance exists across the switch.



The grounded - emitter circuit.

Figure 2.

c. An a-c voltage is applied, because of inter-electrode capacitance.

6. Some switches, thyratrons, for example, cannot be turned off by simply reversing the turn-on procedure.

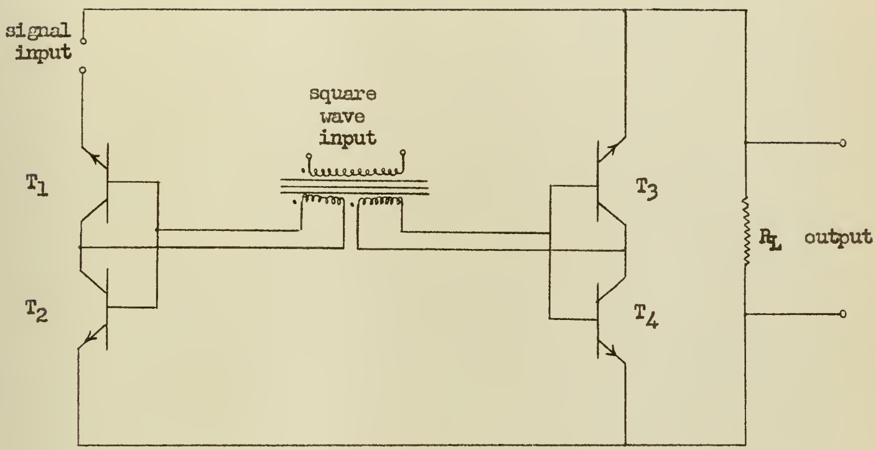
7. Other switches, diodes, for example, will not conduct a current equally well in either direction.

Although this listing is incomplete, it does indicate the general nature of the problems one encounters in choosing the right switch for the job.

It has been found that junction transistors are very good switches. They can be made to approach the ideal switch more closely in many respects than any other known device (3). Within its current, voltage, power and temperature limits, a junction transistor switch has useful properties. Some of these may be shown with the aid of the grounded-emitter circuit of Fig. (2).

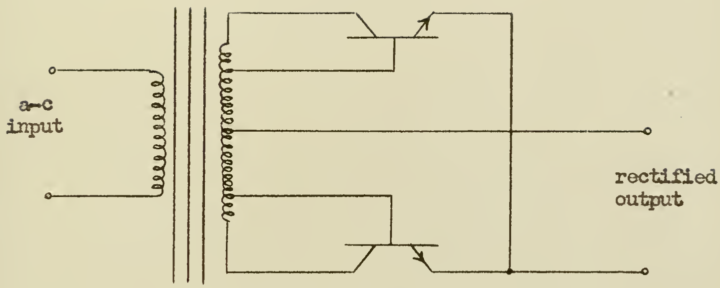
When base current, I_b, is flowing in the direction indicated, the transistor is in its "on" state. V_{ce} is, typically, less than five millivolts and the resistance between collector and emitter, r_{ce}, is usually between one and five ohms. I_c may flow either from collector to emitter or from emitter to collector equally well, within certain magnitude limits which depend on the value of I_b.

When I_b is made to flow in a direction opposite to that shown in Fig. (2) so that the magnitude of V_{be} is 0.1 volts or greater, the transistor is driven to its "off" state. In germanium transistors under this condition, I_c is a few microamperes and r_{ce}, several megohms. Silicon units usually fall in the region where I_c is measured in millimicroamperes and r_{ce}, in tens and hundreds of megohms.



A transistorized chopper circuit.

Figure 3.



An efficient full-wave rectifier

Figure 4.

A transistor has other useful properties. For example, when it is used to replace a mechanical switch, these are some of the advantages obtained:

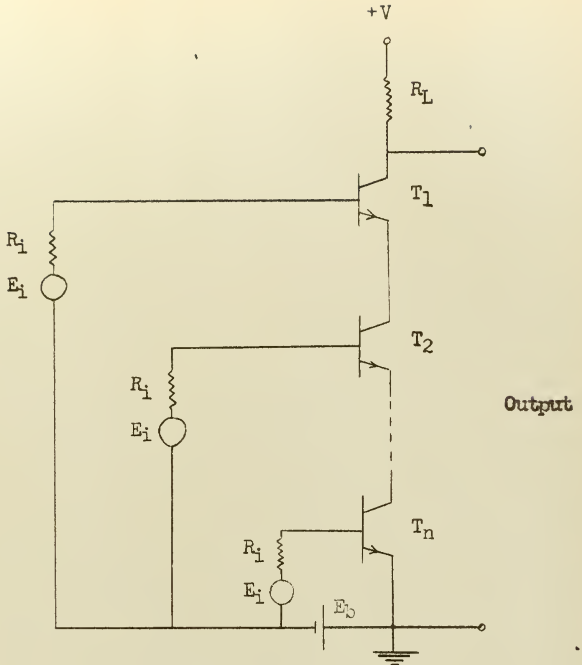
1. A transistor has no contacts or moving parts to wear out.

2. There is no problem of arcing, although voltage breakdown problems do exist.

3. Transistor switches may be operated much faster than mechanical switches; for example, a transistorized chopper, such as is shown in Fig.(3), will operate easily at a rate of ten kilocycles, a rate at which it is nearly impossible for a mechanical chopper to operate. Transistorized choppers do not have appreciable phase shift in the audio range and can be driven equally well at any of a wide range of chopping frequencies. Mechanical choppers, on the other hand, suffer from phase shifts and mechanical resonances, and they are usually designed for operation over a relatively narrow range of chopping frequencies, (6).

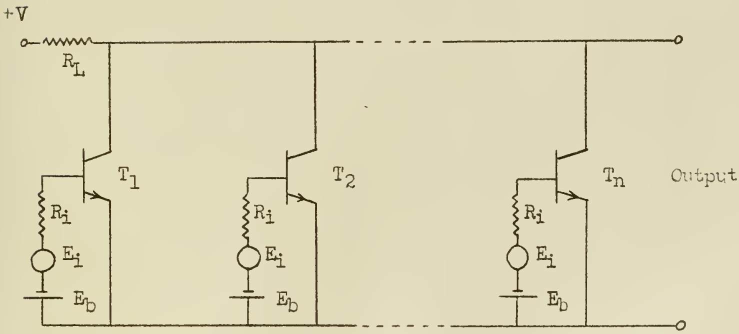
4. Transistors can be designed to operate stably under severe mechanical shock, while mechanical switches are more difficult to design for this kind of service.

Other useful properties of the transistor can be shown from the advantages which are obtained when it is used to replace a diode. Fig. (4) shows a full-wave transistorized rectifier which, to achieve a higher efficiency, replaces a diode rectifier. In this application of the transistor, advantage is taken of the high front to back ratio, about 10^5, which a transistor can exhibit for voltages down to about one millivolt (1). The transistor possesses other advantages over a diode when it is used as a non-regenerative switch in digital computer logic circuits to replace



(A)

Example of an "and" logic circuit using junction transistors



(B)

Example of an "or" logic circuit using junction transistors

Figure 5

somewhat more awkward diode switching circuits. Examples of series transistor "and" and "or" circuits are shown in Fig. (5). A diode logic circuit suffers from loading effects when several logic levels are cascaded. This loading limits the number of levels that can be cascaded to from two to about four without intermediate buffering (3). The transistor logic circuits overcome this difficulty because a transistor has a current gain from its base to its collector.

Despite their many virtues transistor switches have several weaknesses:

When a transistor is in its "on" state and even when I_c is zero and no external collector voltage is applied, there is still a small voltage of about one to five millivolts between collector and emitter of a typical transistor. This voltage is present whenever base current is flowing and in some cases it is a considerable inconvenience. For example, in Fig. (3), transistors in the pairs T_1, T_2 and T_3, T_4 must be nearly identical in order to cancel the signal that this residual voltage would produce at the chopper output.

When the transistor is in its "off" state and collector voltage is applied, there is a small leakage current which is a few microamperes for germanium transistors and a few millimicroamperes for silicon transistors. This leakage current can be troublesome when switching is attempted in high impedance circuits.

The collector to emitter capacitance and the transit time of minority carriers across the base region of a transistor will, in some cases, be large enough to impair its usefulness in circuits where fast switching action is desired (9). This aspect of transistor switches is not discussed in this paper; it is mentioned only because of the

limitations it places on the usefulness of transistor switches.

The current that a transistor can switch is limited in at least two ways. When the base is current saturated, the maximum value of I_c is limited by the heat which it is allowable to dissipate in the transistor. The base can be driven out of current saturation by making I_c too large; then the power dissipation increases undesirably. Any increase in power dissipation is generally unacceptable because it causes an increase in temperature, a change in transistor parameters (Equation 1-11 shows that the transistor is temperature sensitive.) and, eventually, if it is not checked, a burn-out of the transistor.

The maximum collector voltage that a transistor will withstand in its "off" state depends on the voltage breakdown mechanisms which have been discussed at some length in Chapter I. When a voltage breakdown occurs, collector current usually increases and the transistor may be lost because of overheating at one of its junctions.

The preceding few paragraphs have pointed out some of the advantages and weaknesses of practical transistor switches. In choosing between a transistor and some other device to perform a switching function, the relative merits of the two devices must be considered and to determine what may be expected of the transistor, one must either be able to compute its behavior from commonly available small signal parameters using, for example, the equations developed in Chapter I or he must be able to measure the parameters of a transistor in the conditions under which it is to be used. Chapter III chooses the latter alternative. It describes a device for measuring the static switching properties of the transistor.

CHAPTER III

A DEVICE FOR MEASURING THE STATIC SWITCHING PROPERTIES OF JUNCTION TRANSISTORS

It is indicated in Chapter I that analytical methods of determining the static switching properties of a transistor are subject to disadvantages. An alternative to these methods is the actual measurement of the desired parameter. The subject of this chapter is the development of circuitry to make these measurements. Capabilities which might be desired of such circuitry are:

1. An easily interpreted dynamic display of information.

2. Simple and quick measuring procedure.

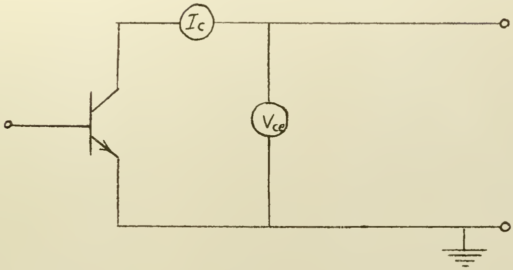
3. Sufficient versatility to permit measurement of all commonly encountered types of junction transistors.

Perhaps the simplest and most easily understood display of information about a transistor is the static collector characteristic. When this characteristic is extended through regions I, II and III, so as to show detailed information at the origin, it displays nearly all the information one needs in order to assess the static switching performance of a transistor. Furthermore, in contrast to the meter display, it has the advantage of showing information about a large number of operating points, while a meter is usually able to display information about only one operating point at a time.

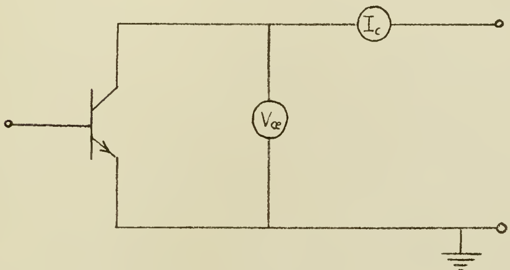
A plot of the static collector characteristic of a transistor is simply a plot of V_{ce} and I_c for various known values of I_b or V_{be}. There are several possible arrangements of current and voltage measuring devices which will accomplish this purpose and these are shown in Fig. (6) where

Possible arrangements of voltmeter and ammeter for measuring collector current and collector voltage.

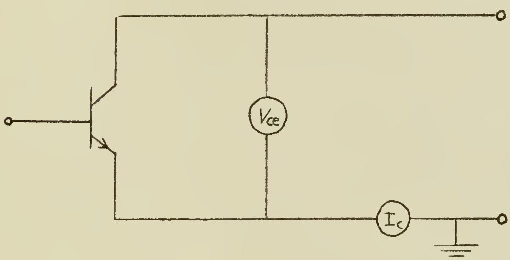
A.



B.



C.



D.

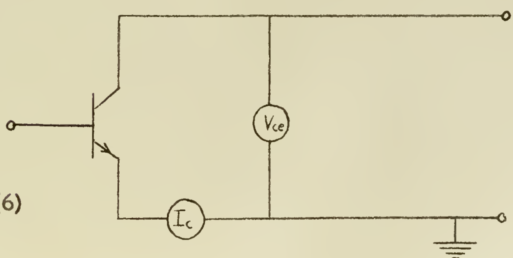


Figure (6)

the measuring device is shown merely as an ammeter or a voltmeter.

In determining which of these circuits is most desirable, the "off" condition of the transistor will be considered first. In this state r_{ce} is high; it is perhaps as much as 200 megohms. I_c is small; as little as 10 millimicroamperes and V_{ce} may be any value up to about 50 volts, depending on the breakdown voltage of the particular transistor under test. Circuits B and C have the disadvantage of requiring that the current through the voltmeter be subtracted from the ammeter reading. Because the current through the transistor will probably be much less than the current through the voltmeter, the circuits B and C are not attractive. Circuit A requires that the ammeter be isolated from ground. The dynamic measurement of ten millimicroamperes with a device such as an amplifier poses serious noise and leakage problems; for this reason, a grounded ammeter such as is shown in circuit D is attractive. Furthermore, small current amplifiers of the grounded type have been built to measure currents in the micromicroampere range, (14). For these reasons circuit D was chosen for the basic amplifier configuration for the "off" condition.

The "on" condition is considered next. In this state r_{ce} is very low; usually it is less than 10 ohms and V_{ce} is, typically, less than five millivolts. Under these conditions, the voltage drop across an ammeter is likely to be of comparable magnitude to the voltage drop across the transistor; circuits A and D are, therefore, eliminated. If the current source has high impedance compared to r_{ce}, then to a good approximation, I_c is independent of V_{ce} and need not be measured directly. For these conditions circuit B seems most attractive and, therefore, it was chosen for the basic configuration in the "on" condition.

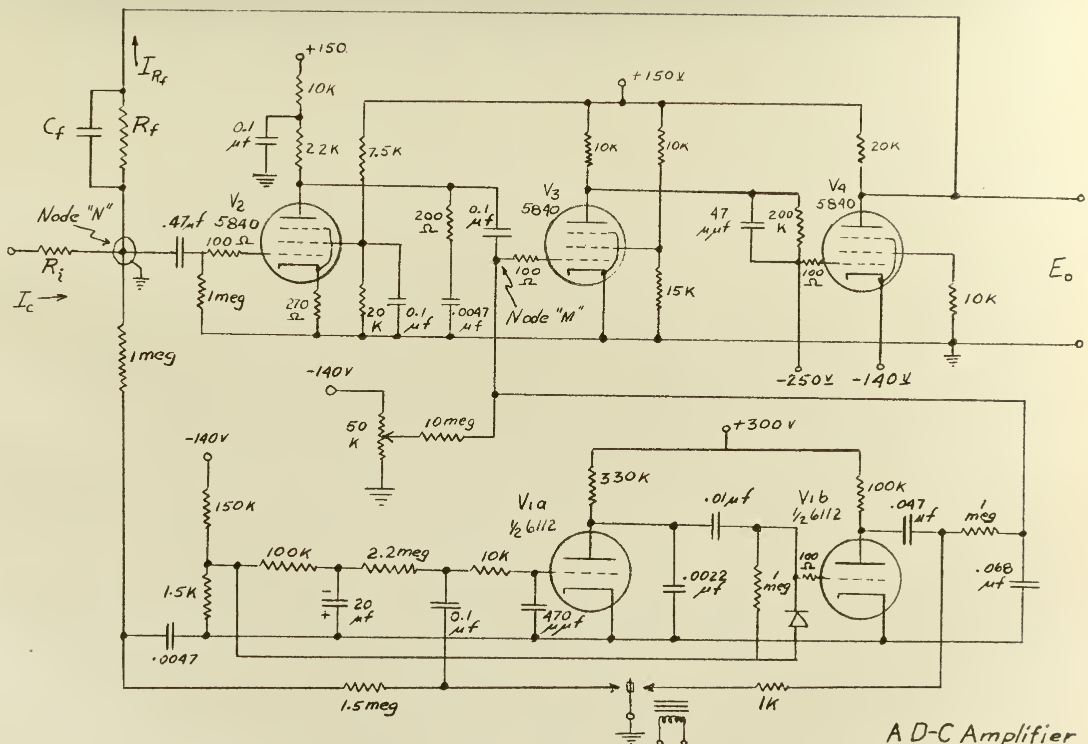


Figure (7)

A D-C Amplifier Possessing low input Resistance

There are at least two methods of automatically displaying the collector characteristic; one of these is by use of a servo-plotter. In this arrangement, the x-axis amplifier of the plotter is fed a voltage proportional to V_{ce}. Either I_b or V_{be} is then held constant and the characteristic is traced by varying either V_{ce} or I_c arbitrarily. Because a servo-system, typically, has only a few cycles per second bandwidth, it can be used to plot voltages which would be masked by noise in a wider band device. The servo-plotter, however, is slower than a purely electronic device; furthermore, the plotting paper must be continually renewed.

Another method of displaying the collector characteristic uses a cathode-ray oscilloscope instead of a servo-plotter. In this case voltages proportional to V_{ce} and I_c may be read directly from a gridded template covering the cathode-ray tube face. This method of displaying the collector characteristic is best suited to a quick check device and, hence, it was the one chosen.

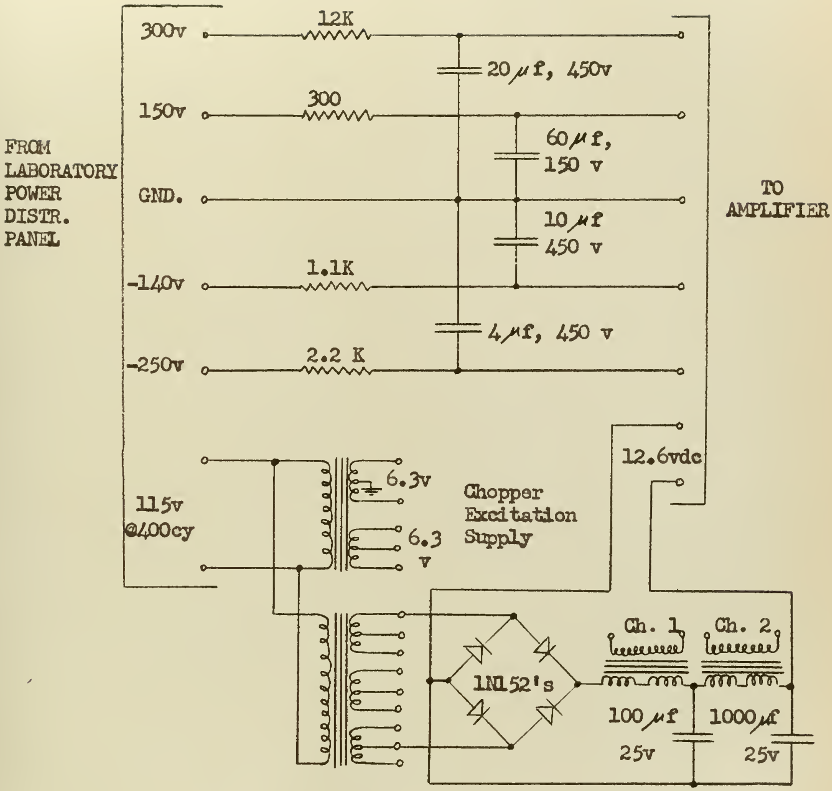
It has already been indicated that rather small values of voltage and current must be measured. At the time this problem arose, there was no available direct-coupled oscilloscope that possessed enough gain to make these measurements; furthermore, the high input impedance of an oscilloscope makes it a poor current sink. It is necessary, then, that an amplifier precede the oscilloscope. This amplifier must be direct-coupled; it must have very low input impedance, high gain, negligible drift, low internal noise and moderate bandwidth. An amplifier was found at Hughes Aircraft Company which met these requirements very well. A schematic of this amplifier is shown in Fig. (7). Operation of the circuit is as follows:

The sum of currents into node N must equal zero. The voltage of node N is maintained constant, for practical purposes, by a very large amount of negative feedback from the amplifier output through R_f. If E_o is adjusted to ground potential and no current is supplied to the node externally, then I_{R_f} must be zero and node N must be at ground potential. As soon as an external current, I_c, is fed to node N, an equal current must be removed from the node through R_f; to accomplish this, $E_o = \frac{I_{R_f}}{R_f} = -\frac{I_c}{R_f}$, and so E_o is proportional to I_c.

A high degree of d-c stability is achieved in this amplifier by use of chopper stabilization. The means by which stabilization is obtained is as follows:

Referring again to Fig. (7), it is seen that the coupling to the input of V_2 is a high-pass filter with a corner frequency of about 0.3 cycles per second while the coupling to the input of V_1 is a low-pass filter with a corner frequency of about 30 cycles per second so that the frequency spectrum of the input signal is divided between V_1 and V_2 with some overlapping. The signal passing through V_2 is amplified and inverted. The signal to V_1 is chopped at a 400 cycle per second rate, amplified as an a-c signal and then synchronously rectified at the output of V_1. This rectified signal is passed through another low-pass filter with a corner frequency of about two cycles per second and then summed at node M with the signal from V_2. The composite signal is further amplified by V_3 and V_4 and a portion of it is fed back through R_f as previously described. The over-all amplification, without feedback, is about 15,000.

In order to reduce the noise level in this amplifier, tube heater circuitry of the original amplifier was modified to operate on direct current. All 400 cycle wiring was removed from the amplifier chassis;

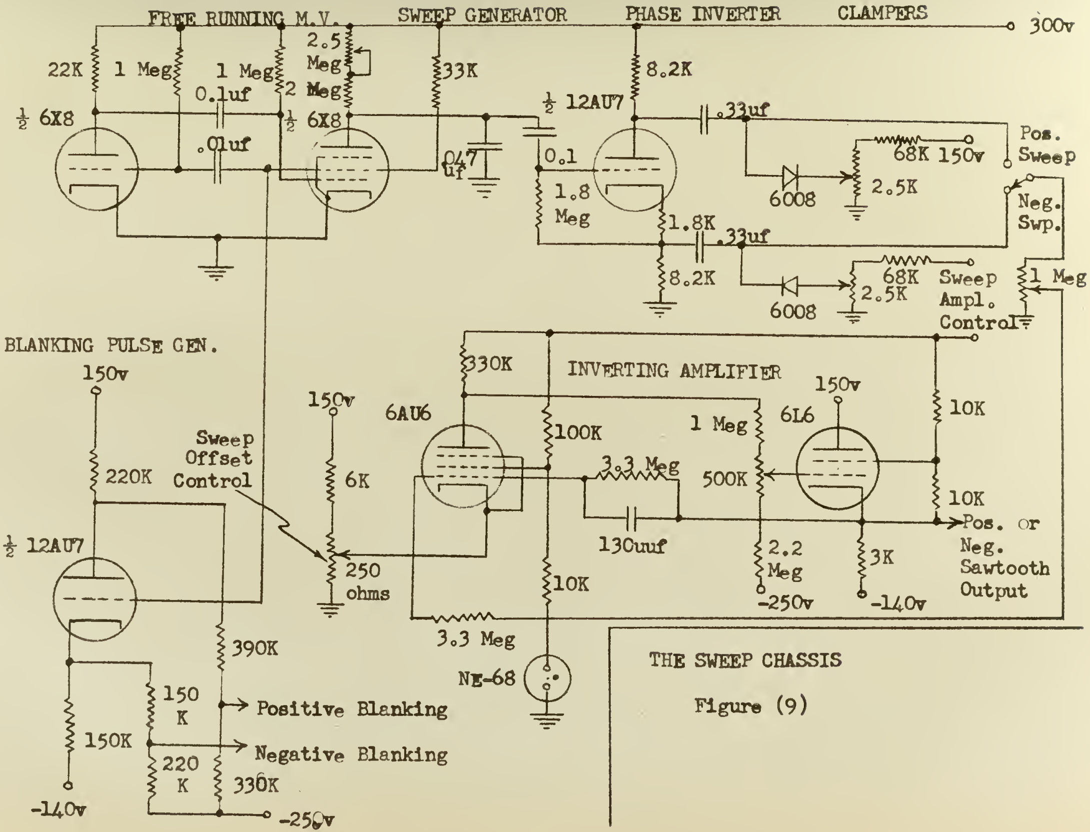


AMPLIFIER POWER SUPPLY

Figure (8)

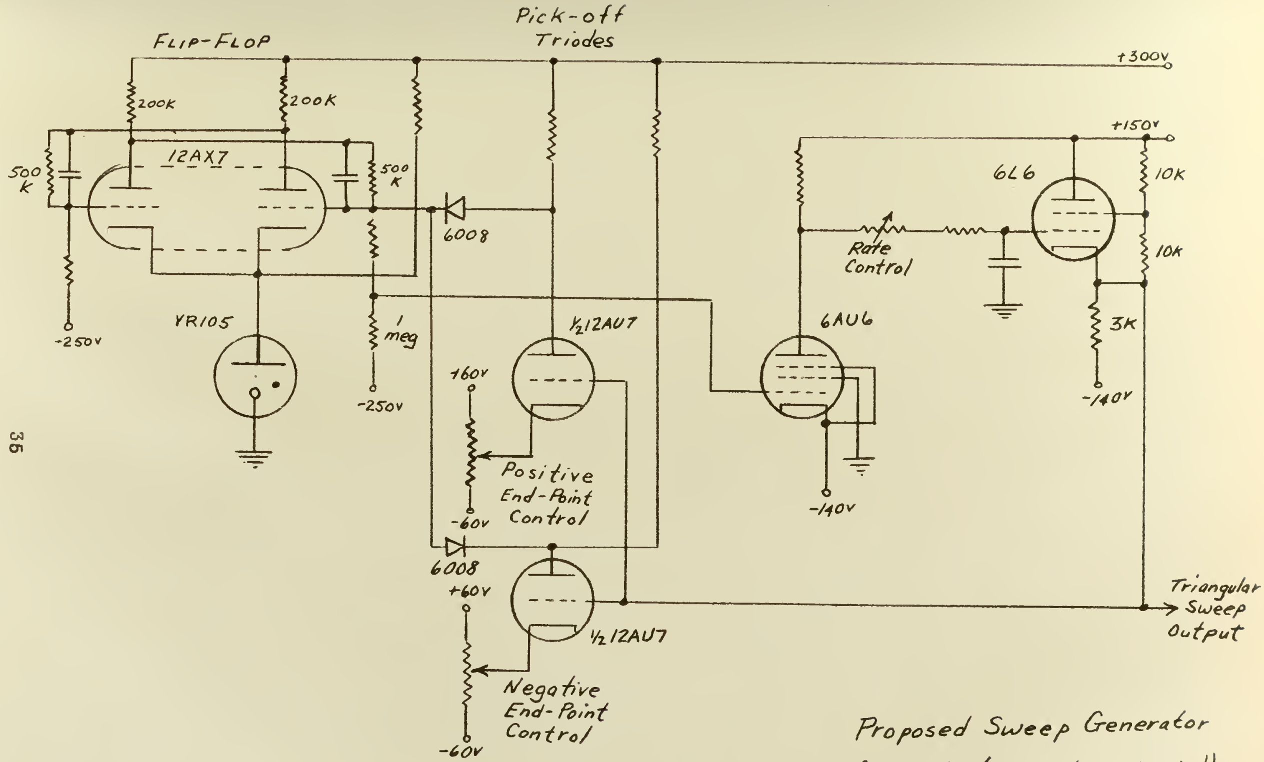
the chopper was supplied with 400 cycle power through separate shielded leads. The mechanical vibration from the chopper was prevented from exciting microphonic noise in the tubes by mechanically isolating the chopper and electrically connecting it to the amplifier with three inch leads of very flexible wire. All d-c power, obtained from a general distribution system in the laboratory, was run through low-pass decoupling filters before it was fed to the amplifier. Fig. (8) shows the power supply chassis. Finally, the entire amplifier was placed inside a grounded aluminum box in order to reduce stray pick-up noise. When these precautions had been taken, noise at the amplifier output was about three millivolts peak to peak. It was found that this level could be further decreased by reducing the amplifier bandwidth. With a 690 micromicro-farad capacitor shunted across R_f, the output noise was not observable on a Dumont 304-H oscilloscope set at maximum gain, about 30 millivolts per inch. A "Millivac" d-c voltmeter placed at the amplifier output showed a slight drift. After a one hour warm-up, the amplifier drifted about two millivolts in a period of 30 minutes. It was then found that with no current fed to the amplifier input, E_o was slightly above ground potential. In order to compensate for this condition, a ten megohm resistor was connected to node M, as shown in Fig. (7), to supply an offset current that would bring E_o back to ground potential.

Another requirement of the circuits chosen for displaying the collector characteristic is that a choice of either a swept voltage or a swept current be available to drive the collector of the transistor. It was decided, initially, that the sweep generator should provide either a positive or a negative voltage or current sawtooth, that the origin of the sawtooth should be accurately clamped at ground potential and that



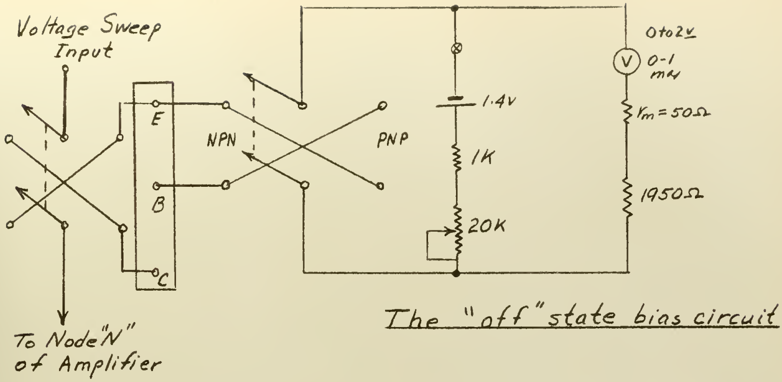
THE SWEEP CHASSIS

Figure (9)



Proposed Sweep Generator
Circuit (was not constructed)

Figure (10)



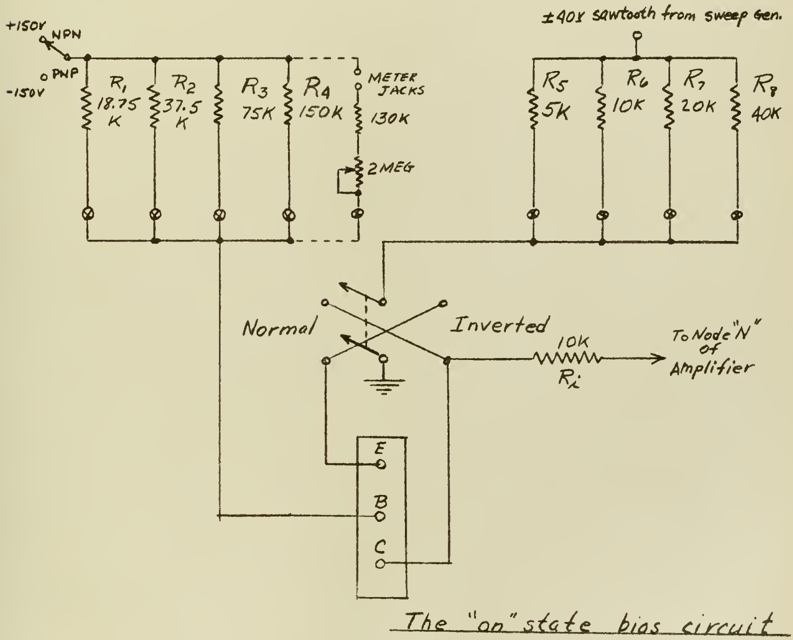


Figure (11). The Simplified Bias Chassis Circuit

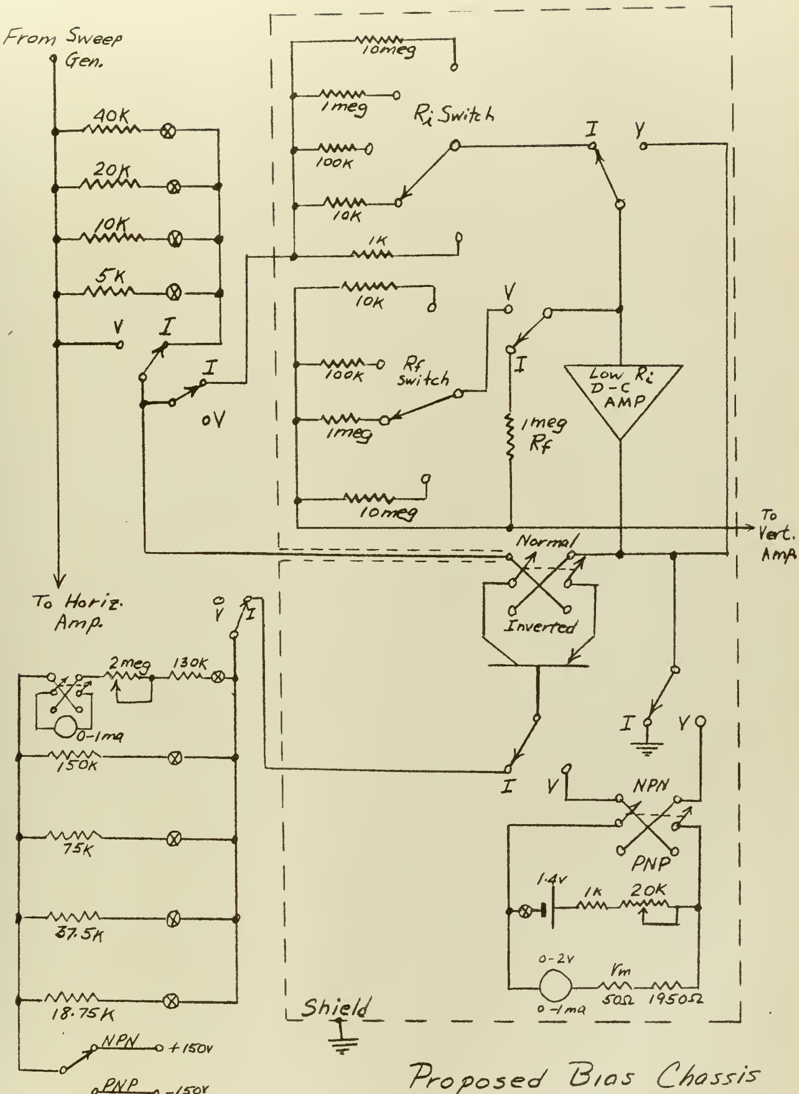


Figure (12) Proposed Bias Chassis Circuit

(Was not constructed)

its other extremity should be variable between limits of zero to forty volts or zero to ten milliamperes to a 3,000 ohm load resistance. Sweep repetition frequency was to be as low as maintaining a fairly persistent trace on the cathode-ray tube face would permit. This frequency was found by trial to be about 20 cycles per second. The rest of the details of the development of the sweep chassis adhere to well established vacuum circuit design techniques and for this reason they are not included here. Fig. (9) shows the sweep circuit as it was finally evolved.

Measurements which were made of several transistors after the entire measuring circuit had been completed and set up showed that it would have been preferable to have made the sweep generator to these specifications:

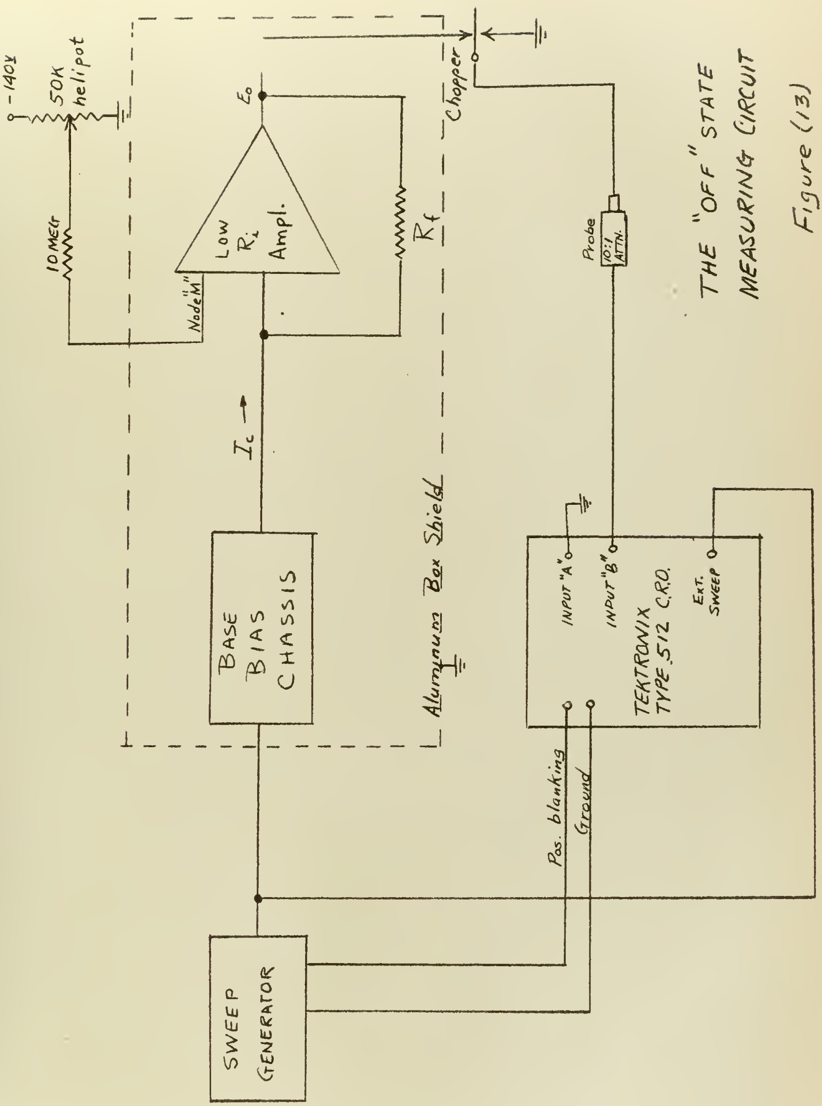
1. Provide a triangular sweep voltage or current with independently controllable end-points each of which is to be continuously variable between $\pm$ 40 volts.

2. Provide for sweep frequencies variable between four volts per second and about 4,000 volts per second.

3. Provide an output amplifier capable of driving a 3,000 ohm load to 15 milliamperes.

A circuit was designed to meet these specifications but because of limitations placed on development time for the project, it could not be constructed or tested. This circuit is shown in Fig. (10).

The final part of the measuring circuit which was designed was the bias chassis, shown in Fig. (12). The function of this chassis was to provide the needed switching functions for bias and sweep voltages and currents, and to provide a holder for the transistor under test. This chassis was not constructed because of time limitations, although a simplified "haywire" version of this chassis, shown in Fig. (11), was



THE "OFF" STATE MEASURING CIRCUIT

Figure (13)

constructed to make the measurements which are presented in Chapter IV. The simplified bias chassis which was actually used, provided a one volt base back-bias that kept the transistor cut off for the "off" state measurements. For the "on" state measurements, base currents were obtained from a current ladder consisting of R_1 through R_4 shown in Fig.(11). This ladder was arranged to provide binary values of current so that any value of current from one milliampere to 15 milliamperes was available in one milliampere steps simply by switching in the proper branches of the ladder. Later, another branch was added to the ladder; this branch contained a pair of jacks for a milliammeter, and a potentiometer to vary the branch current. In this way measurements at base currents less than one milliampere were obtained. A similar ladder, consisting of R_5 through R_8 was used to adjust the I_c sweep current. In this case, the ladder was continuously fed either a positive or a negative 40 volt, peak to peak, sawtooth. The magnitude of the I_c sawtooth was set with switches in the branches of the I_c ladder. This ladder operated to provide I_c sweep amplitude variations in one milliampere steps in a manner similar to that described for the I_b ladder.

When the separate parts of the measuring circuit are interconnected to make "off" state measurements, the circuit shown in Fig. (13) is obtained. R_f, in this circuit, is adjusted to a value appropriate to the expected magnitude of I_c. This adjustment of values is made by soldering the desired resistor into the circuit.

"Off" state circuit calibration:

Supposing that I_c= 5 microamperes is expected, R_f is chosen to produce a moderate amplifier output voltage, for example,

$$E_o = -I_c R_f = I_{R_f} R_f = 5\mu a \times 100 K\Omega = 0.5 \text{ volt}$$

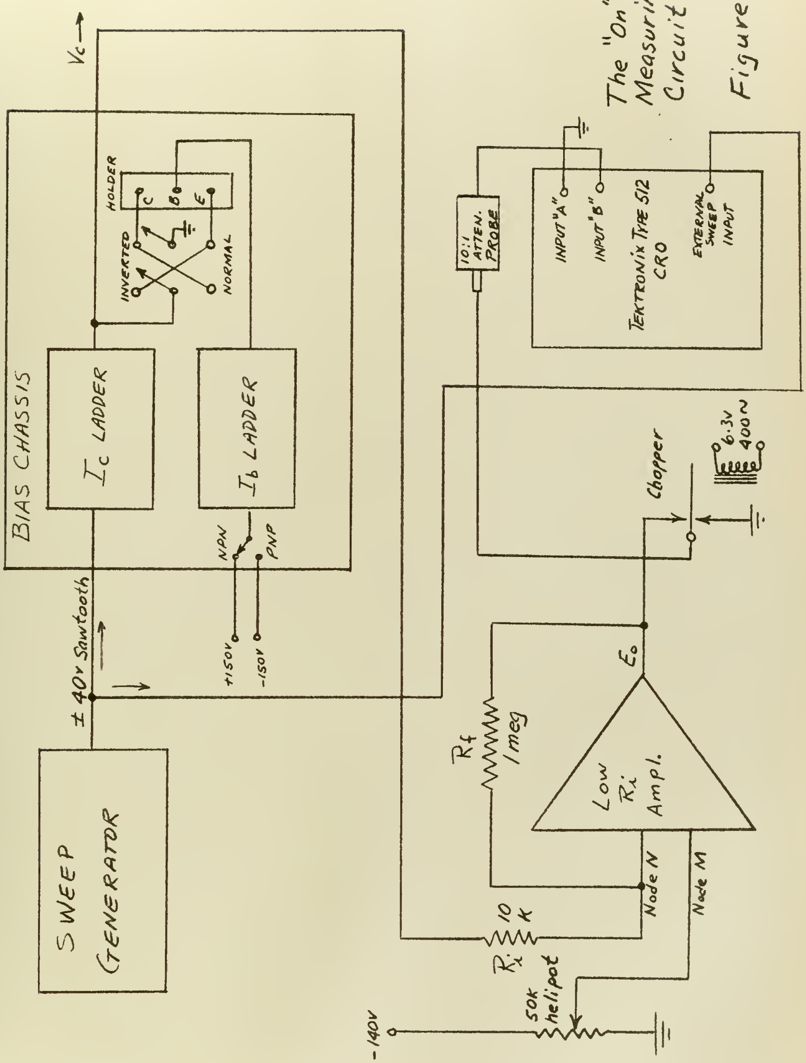
which calibrates the amplifier so that, $I_c/E_o = 10\mu a/volt$. The x-axis amplifier gain is then adjusted using the internal calibrating circuitry of the oscilloscope. This gain is set to produce a known displacement of the trace for an input voltage of 0.5 volts. Supposing that the deflection sensitivity is chosen at 0.1 volts/cm, the x-axis then is calibrated to read,

$$I_c = 10\mu a/volt \times 0.1\ volt/cm = 1\mu a/cm.$$

Internal calibrating circuitry in the oscilloscope or, in the absence of such circuitry, some known external voltage is used to set the horizontal deflection sensitivity so that an easily read scale for V_{ce} is produced which will also permit the sweep length of V_{ce} to occupy most of the horizontal space on the cathode-ray tube face. Base bias should be set to plus one volt for a p-n-p transistor or to minus one volt for a n-p-n transistor. V_{ce} should be switched to provide a positive sweep for an n-p-n transistor or a negative sweep for a p-n-p transistor. Sweep origin should be checked to be sure it is at ground potential, although the adjustment will hold fairly well after the sweep amplifier has warmed up for about half an hour. The sweep offset control is used to make this adjustment; see Fig. (9). Sweep amplitude should be set to zero.

When these adjustments have been made, the transistor may be placed in its holder and the desired "off" state collector characteristic will be displayed on the calibrated cathode-ray tube face as the sweep amplitude control is advanced.

To make measurements in the "on" state, a rearrangement of the circuitry is required, as may be seen by comparing configurations B and D



The "On" State Measuring Circuit

Figure (14)

of Fig. (6). In the "on" state circuit, Fig. (14), the chopper stabilized amplifier is used as a voltage amplifier. The amplifier still operates to keep node N at ground potential; however, input current now flows through R_i, a fixed precision resistor, and is therefore proportional to the input voltage, V_{ce}. Aside from this difference, the amplifier functions just as it did when it was used to amplify I_c in the "off" state. Amplifier voltage gain is, for all practical purposes, equal to the ratio R_f/R_i, while amplifier input resistance equals R_i. Calibration procedure for this circuit is as follows:

Before placing the transistor in the holder, all switches in the I_b and I_c ladders should be switched off. Set R_f and R_i to one megohm and 10,000 ohms, respectively. This establishes amplifier gain at 100, which was found to be a practical value for all transistors that were measured. Set the horizontal sweep control on the oscilloscope to produce some convenient length, say five cm. Check the I_b source to see that it is correctly polarized; positive for n-p-n transistors and negative for p-n-p transistors. Switch in a moderate value of I_b; this value must be determined from a knowledge of the transistor. One to five milliamperes was found to be safe for the junction transistors which were tested. I_c may be polarized either positively or negatively. Switch in a moderate value of I_c, say five milliamperes. A trace will be observed above the chopped ground reference line and at an angle to it. Adjust the oscilloscope's x-axis amplifier gain to produce a convenient displacement and then check the gain calibration using the internal calibration circuitry of the oscilloscope. The vertical calibration of the display = 100 x oscilloscope gain. The physical length of the horizontal sweep will remain constant but the y-axis calibration will be the total value of current

set on the I_c ladder divided by this fixed physical length of the I_c sweep.

It will be seen by referring to Fig. (13) and Fig. (14), that a chopper is placed at the x-axis input of the oscilloscope. This chopper commutates the x-axis voltage between ground potential and the potential at the output of the amplifier, thereby assuring that ground potential is always accurately located on the cathode-ray tube face. This device does not eliminate the need for a d-c coupled oscilloscope but it does eliminate, to a large extent, the error which would otherwise be introduced by a drift in the x-axis amplifier of the oscilloscope. The y-axis amplifier input is not chopped for two reasons. First, the choppers would have to be operated either with fixed phase difference so that the beam would have to be time shared equally between the x-axis, y-axis and the desired curve or they would have to be operated at different frequencies. Either of these alternatives is awkward; the first requires that a phasing network be introduced into the a-c supply for one of the choppers and the second requires introducing another a-c power source. Second, the drift in the y-axis amplifier has been found to be negligibly small anyway. The y-axis amplifier is always fed the independent variable, I_c for the "on" state and V_{ce} for the "off" state. This signal level is always great enough so that only a moderate gain is needed in the y-axis channel.

CHAPTER IV

MEASUREMENTS AND CONCLUSIONS

To check the operation of the device described in Chapter III, it was used to measure the collector characteristic curves of several transistors. The parameters, α_i, α_n, I_{eo} and I_{co}, of these same transistors were measured and were used to compute the collector characteristic curves. The computed curves and the measured curves were then compared. One of these comparison runs, which was made using a Sylvania 2N94A alloyed-junction transistor, has been chosen for detailed discussion of this procedure. The inverted configuration was chosen for this run because a preliminary check showed that I_e was somewhat less than I_c in its "off" state. The measuring circuit which was used is shown in Fig. (13). The procedure described in Chapter III was followed in calibrating and setting up the testing circuitry. The cathode-ray tube presentation which was obtained is shown in Fig. (15).

The following information about this transistor can be obtained from this display:

$$r_{ec} = \frac{(25-5)\text{ volts}}{(4-3.6)\,\mu a} = 50 \text{ megohms}$$

$$R_{ec} = \frac{10 \text{ volts}}{3.7\,\mu a} = 2.7 \text{ megohms}$$

$$\text{Current offset}\Big|_{V_{ec}=0} = 3.2\,\mu a$$

Emitter voltage breakdown is noticeable above $V_{ec} = 30$ volts.

V_{ec}	I_e
0v	3.2 μa
5	3.6
10	3.7
15	3.8
20	3.9
25	4.0
30	4.2
35	4.8
37	5.25

"Off" State Collector Characteristic
Plot for an inverted type 2N94A
Alloyed Junction Transistor

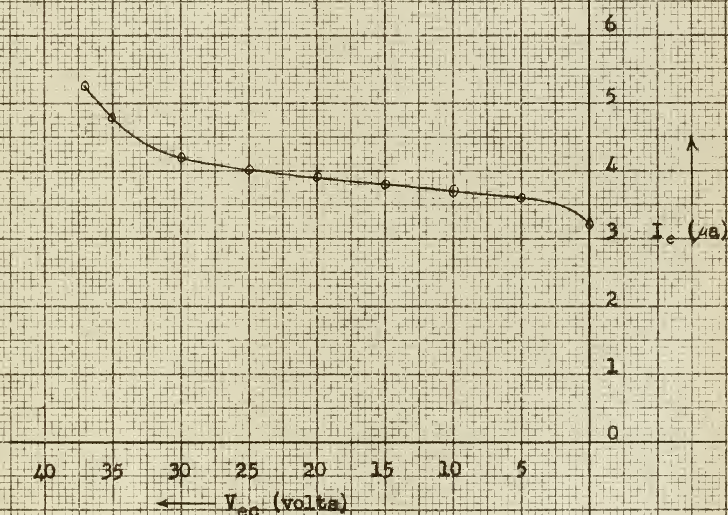


Figure (15)

One may also find the offset current by measuring small signal parameters, $I_{co}, \alpha_n, \alpha_i$. These values can be substituted into equations (1-10) and (1-11) to arrive at the same information as was measured directly. Since it is rather difficult to measure α_n and α_i when the transistor is cut-off, values will be arbitrarily chosen from Region II measurements at the point where $I_b = 35\mu a$, $I_e = 1.0$ ma, and $I_c = .965$ ma. Under these conditions, $\alpha_n = .985$, $\alpha_i = .965$; see Fig. (19). I_{co} was measured at $9\mu a$. Equation (1-10) is used because the collector characteristic plot of Fig. (15) describes the transistor in its inverted condition and as far as the markings on the transistor are concerned it is really a V_{ec} vs. I_e plot. Using the equation,

$$I_e = \frac{1}{1-\alpha_n\alpha_i}\left[-I_{eo}\left(e^{\frac{q\varphi_e}{kT}}-1\right) + \alpha_i I_{co}\left(e^{\frac{q\varphi_c}{kT}}-1\right)\right] \qquad \text{(1-10)}$$

and equation (1-9), rearranged so that

$$I_{eo} = \frac{\alpha_i}{\alpha_n} I_{co}$$

one may compute I_e as follows:

$$I_e = -\frac{\frac{.965}{.985}\times 9\mu a}{1-.965\times.985}\left(e^{-\frac{1}{.026}}-1\right) + \frac{.965\times 9\mu a}{1-.965\times.985}\left(e^{-\frac{10}{.026}}-1\right)$$

where,

$$\varphi_e = V_{be} = -1 \text{ volt}$$

$$\varphi_c = V_{bc} = -10 \text{ volts}$$

Since the exponential terms are negligibly small, they may be dropped, then,

$$I_e = \frac{\frac{.965}{.985}-.965}{1-.965\times.985}\times 9\mu a = \frac{.015}{.05}\times 9\mu a = 2.7\mu a$$

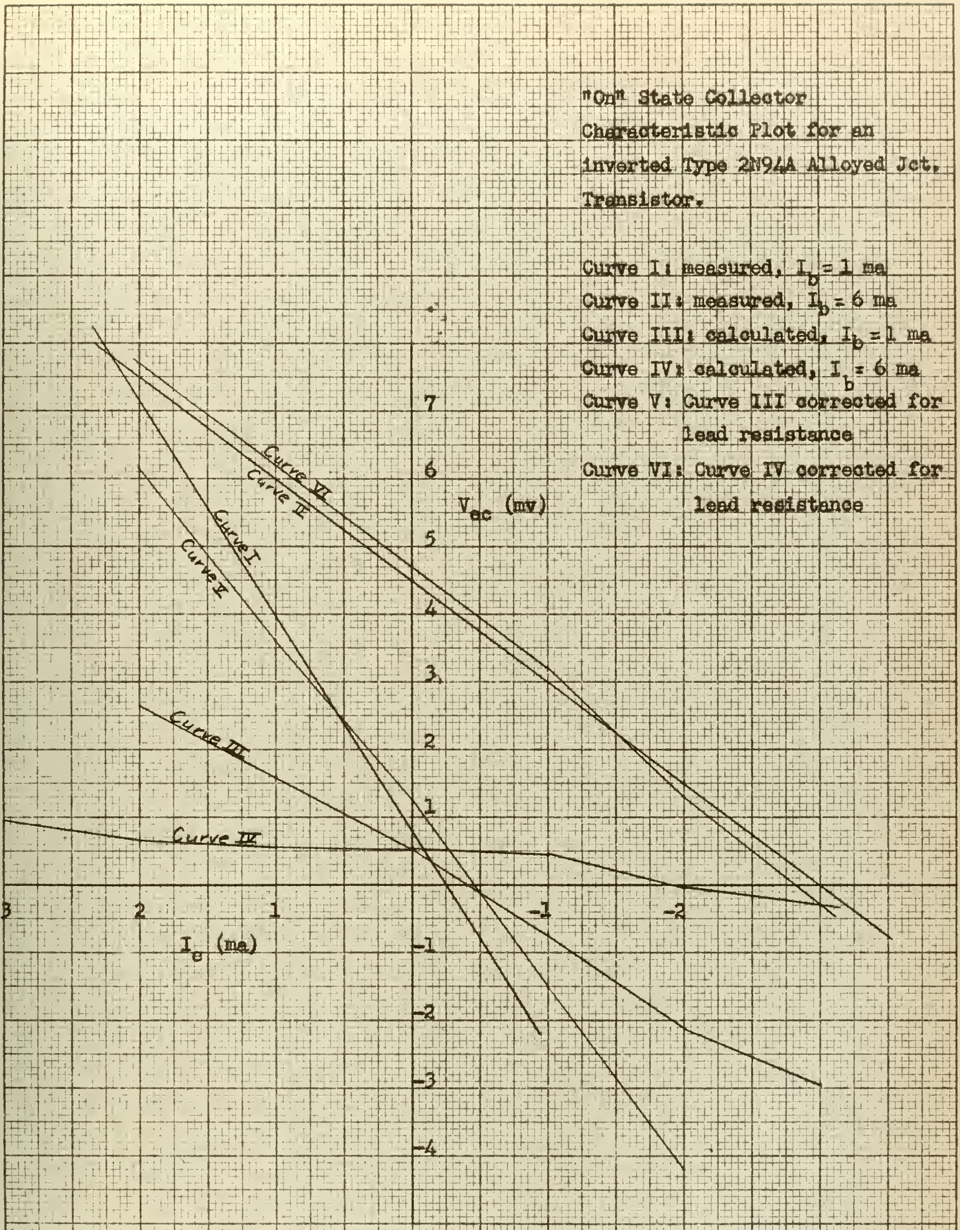

"On" State Collector Characteristic Plot for an inverted Type 2N94A Alloyed Jct. Transistor.
Curve I: measured, $I_b = 1$ ma
Curve II: measured, $I_b = 6$ ma
Curve III: calculated, $I_b = 1$ ma
Curve IV: calculated, $I_b = 6$ ma
Curve V: Curve III corrected for lead resistance
Curve VI: Curve IV corrected for lead resistance
V_{ec} (mv)
I_e (ma)
Curve I
Curve II
Curve III
Curve IV
Curve V
Curve VI
7
6
5
4
3
2
1
-1
-2
-3
-4
3
2
1
-1
-2

Figure (16)

This value compares more favorably with the measured value of $3.2\mu a$ than is apparent at first glance because the computed quantity does not account for any collector leakage resistance in shunt with the collector junction.

In the above calculation for I_e, when φ_e and φ_c are less than a few tenths of a volt, the exponential terms become negligibly small; I_e and I_c are then nearly independent of variations in emitter and collector voltage. When this is so, r_{ec} must be very large; greater, in fact, than the leakage resistance shunting it. Any calculation which does not include this leakage resistance will give a misleading value of r_{ec}.

R_{ec} is found from equation (1-17),

$$R_{ec} = \frac{V_{ec}}{I_e}$$

and at V_{ec} 10 volts,

$$R_{ec} = \frac{10\,v}{2.7\mu a} = 3.7 \text{ megohms}$$

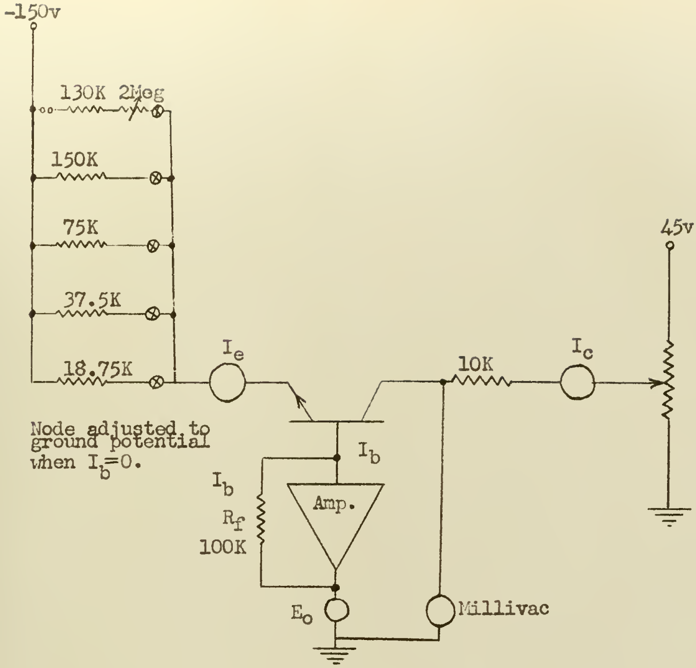
a value which compares reasonably well with the measured value of 2.7 megohms.

The "on" state measurements for this transistor are shown in Fig.(16) as Curves I and II. The measuring circuit which was used is shown in Fig. (14); the procedure which was followed in making these measurements has already been stated in Chapter III. Fig. (16) will be discussed since this figure shows the "on" state behavior of the same 2N94A transistor as was just described in its "off" state. The useful information which can be obtained from these curves is as follows:

For curve I with a base drive current of 1 ma,

Voltage offset = 0.8 mv, when I_e=0.

$$r_{ec} = \frac{0.8\,mv}{0.25\,ma} = 3.2 \text{ ohms}$$



Measuring procedure:

1. Set I_e to desired value.
2. Set V_{cb} to zero on Millivac (25mv scale).
3. Record:
   a. E_o
   b. I_e

Circuit which was used to measure α_n and α_i.

Figure (17)

Data and computations for d-c short circuit current gain, $\propto_n$ and $\propto_i$ of a 2N94A transistor:

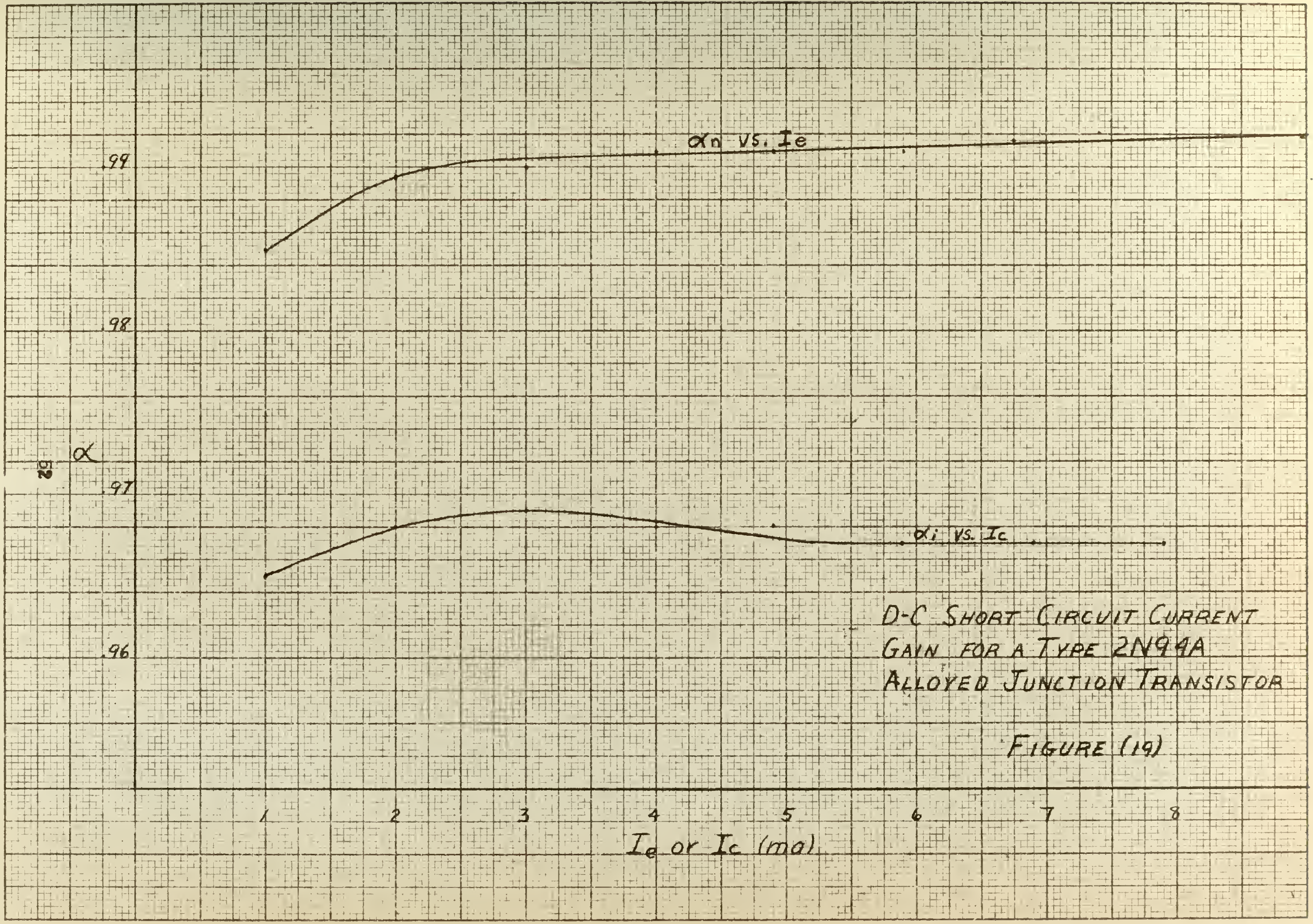
Normal

I_e	E_o	$I_b = \frac{E_o}{100K\Omega}$	$I_e - I_b = I_c$	$\propto_n$
1.0 ma	1.5 volts	15 μa	0.985	.985
2.0	2.25	22.5	1.975	.9887
3.0	3.0	30	2.97	.990
4.0	3.7	37	4.865	.991
4.9	4.4	44	5.85	.991
5.9	5.0	50	6.841	.991
6.9	5.7	57	6.843	.992
7.9	6.55	65	7.835	.992
8.95	7.2	72	8.878	.992
9.9	8.0	80	9.82	.992

Inverted

1.0	3.5	35	9.65	.965
2.0	6.4	64	1.936	.968
3.0	9.3	93	2.907	.969
4.0	12.8	128	3.872	.968
4.9	15.9	159	4.741	.968
5.9	19.2	192	5.708	.967
6.9	23	230	6.670	.967
7.9	26	260	7.640	.967
8.9	29	290	8.610	.968
9.9	31	310	9.590	.968

Figure (18)



D-C Short Circuit Current Gain for a Type 2N94A Alloyed Junction Transistor

Figure (19)

$$R_{ec}\Big|_{I_e = 1 \text{ ma}}$$

This plot also shows the variations of these quantities with respect to changes in I_b. When these curves are extended further than is shown in this plot, a point at which there is a marked increase in r_{ec} is reached. This is the point at which, as I_e is further increased, the fixed value of I_b is unable to keep the transistor saturated and so the transistor operating point passes into either Region II or inverted Region II. The points at which this happens are of interest because they represent the maximum currents which can be conducted through the transistor for any particular base drive current. When this information is combined with the "off" state information, one obtains a fairly complete picture of the static switching properties of the transistor. Before the transistor is used in a switch design problem, information about its dynamic switching behavior should be obtained either from dynamic measurements or from calculations based on small-signal a-c parameters (9).

One can find the same information as is shown in Curve I by using equation (1-13). To use this equation, one must measure α_i and α_n. The measurement of these parameters is based on their definitions,

$$\alpha_n = \frac{I_c}{I_e}\Big|_{V_{cb}=0}$$

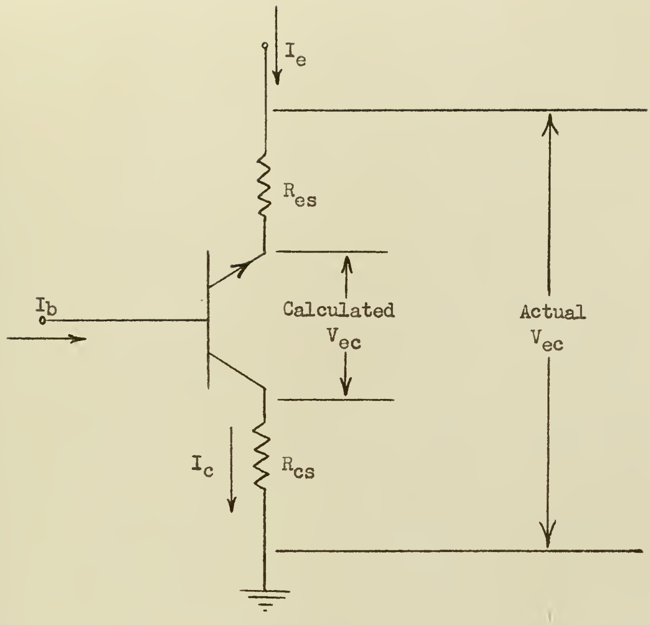
$$\alpha_i = \frac{I_e}{I_c}\Big|_{V_{eb}=0}$$

The circuit shown in Fig. (17) was set up to make these measurements. Data which was obtained is shown in Fig. (18) and is plotted against I_e and I_c in Fig. (19). Solutions for V_{ec} based on equation (1-13) and the

I_c	I_e	I_b	I_e/I_b	α_i	$1-\alpha_i$	$\frac{1-\alpha_i}{\alpha_i}$	$\frac{I_e}{I_b}\frac{1-\alpha_i}{\alpha_i}$	$1-\frac{I_e}{I_b}\frac{1-\alpha_i}{\alpha_i}$	α_m	$1-\alpha_m$	$\frac{I_e}{I_b}(1-\alpha_m)$	$1+\frac{I_e}{I_b}(1-\alpha_m)$	$\frac{1-\frac{I_e}{I_b}\frac{(1-\alpha_i)}{\alpha_i}}{1+\frac{I_e}{I_b}(1-\alpha_m)}$	$\alpha_m\left[\frac{1-\frac{I_e}{I_b}\frac{(1-\alpha_i)}{\alpha_i}}{1+\frac{I_e}{I_b}(1-\alpha_m)}\right]$	$\ln\alpha_m\left[\frac{1-\frac{I_e}{I_b}\frac{(1-\alpha_i)}{\alpha_i}}{1+\frac{I_e}{I_b}(1-\alpha_m)}\right]$	$V_{ec} = .026\ln\left[\frac{1-\frac{I_e}{I_b}\frac{(1-\alpha_i)}{\alpha_i}}{1+\frac{I_e}{I_b}(1-\alpha_m)}\right]$
-1	0	1	0	.965	.035	.0363	0	1	.98	.02	0	1	1	.98	-.0202	.53 mv
-2	1	1	1	.968	.032	.033	.033	.967	.985	.015	.015	1.015	.954	.94	-.062	1.61
-3	2	1	2	.969	.031	.032	.064	.936	.989	.011	.022	1.022	.916	.907	-.1015	2.64
0	-1	1	-1	.968	.032	.033	-.033	1.033	.985	.015	-.015	.985	1.045	1.03	+.0296	-.768
+1	-2	1	-2	.965	.035	.0363	-.0726	1.073	.989	.011	-.022	.987	1.098	1.085	+.082	-2.12
+2	-3	1	-3	.968	.032	.0331	-.0993	1.099	.99	.01	-.03	.97	1.132	1.12	.1135	-2.95
-6	0	6	0	.97	.03	.031	0	1	.98	.02	0	1	1	.98	-.0202	.53
-7	1	6	.167	.97	.03	.031	.00518	.994	.985	.015	.00250	1.0025	.993	.978	-.0222	.576
-8	2	6	.333	.97	.03	.031	.0102	.9898	.989	.011	.00366	1.0037	.989	.975	-.0253	.658
-9	3	6	.500	.97	.03	.031	.0155	.9845	.99	.01	.005	1.005	.983	.966	-.0346	.94
-5	-1	6	-.167	.97	.03	.031	-.00518	1.00518	.985	.015	-.0025	.9975	1.002	.984	-.0166	.42
-4	-2	6	-.333	.97	.03	.031	-.0102	1.0102	.989	.011	-.00366	.996	1.013	1.002	+.002	-.05
-3	-3	6	-.500	.97	.03	.031	-.0155	1.0155	.99	.01	-.005	.995	1.02	1.008	+.008	-.24

Tabular Solutions of Equation (1-13): $V_{ec} = \pm \frac{kT}{q}\left\{\frac{\alpha_m\left[1-\frac{I_e}{I_b}\frac{(1-\alpha_i)}{\alpha_i}\right]}{1+\frac{I_e}{I_b}(1-\alpha_m)}\right\}$

Figure (20)



A circuit showing the current saturated emitter and collector resistances.

Figure (21)

values measured for α_n and α_i are shown in Fig. (20). The calculated values of V_{ec} are plotted in Fig. (16) as Curves III and IV, for $I_b = 1$ ma and 6 ma, respectively. The calculated curves are obviously in error because they have not accounted for the voltage drop through the resistance within the semiconductor regions of the collector and emitter. This resistance may be determined from curves I and II. Fig. (21) shows these resistances which have been designated R_{es} and R_{cs} (8).

$$\text{Corrected } V_{ec} = I_e (R_{es} + R_{cs}) + I_s R_{cs} + \text{Calculated } V_{ec}.$$

At $I_e = 0$, curve I shows $V_{ec} = 4.5$ mv

and curve IV shows $V_{ec} = 0.5$ mv;

therefore the drop across $R_{cs} = 4.5 - 0.5 = 4$ mv.

With $I_c = 6$ ma,

$$R_{cs} = \frac{4\,mv}{6\,ma} = .667 \text{ ohms.}$$

r_{ec} can be found from curve IV as,

$$r_{ec} = \frac{\Delta V_{ec}}{\Delta I_e} = \frac{[.94-(-.24)]\,mv}{[3-(-3)]\,ma} = \frac{1.18}{6} = .197\,\Omega$$

or from small-signal parameters, by using equation (1-15).

V_{ec} can also be found from curve II, as,

$$r_{ec} = \frac{\Delta V_{ec}}{\Delta I_e} = \frac{[4.5-0]\,mv}{[0-(-3)]\,ma} = 1.5\,\Omega$$

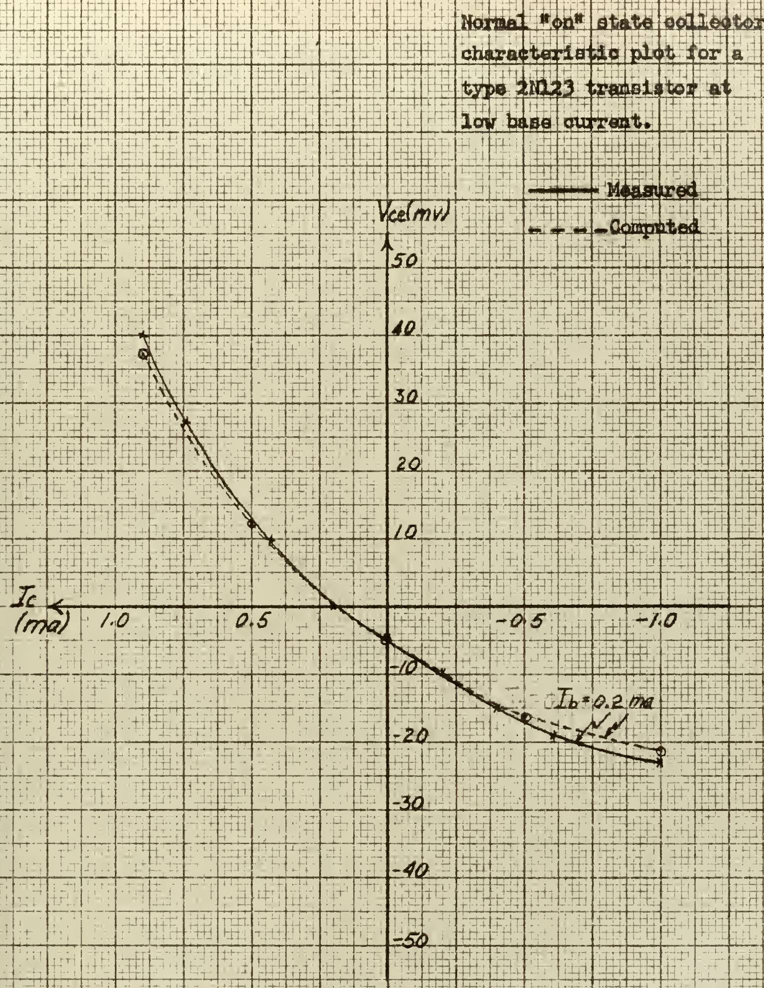
The differences between these resistances must be $R_{es} + R_{cs}$,

$$R_{es} + R_{cs} = \left(r_{ec_{\text{CURVE II}}} - r_{ec_{\text{CURVE IV}}}\right) = 1.5 - 0.2 = 1.3\,\Omega$$

so that,

$$\text{Corrected } V_{ec} = 1.3 I_e + 0.7 I_b + \text{Calculated } V_{ec}$$

When this correction is applied to curve IV, the result is curve VI which agrees very closely with curve II. Curve III, for $I_b = 1$ ma, may similarly be corrected to obtain curve V which is in fair agreement now with curve I.



Normal "on" state collector characteristic plot for a type 2N123 transistor at low base current.
Measured
Computed
Vce(mv)
50
40
30
20
10
-10
-20
-30
-40
-50
Ic (ma)
1.0
0.5
-0.5
-1.0
Ib=0.2 ma

Figure (22)

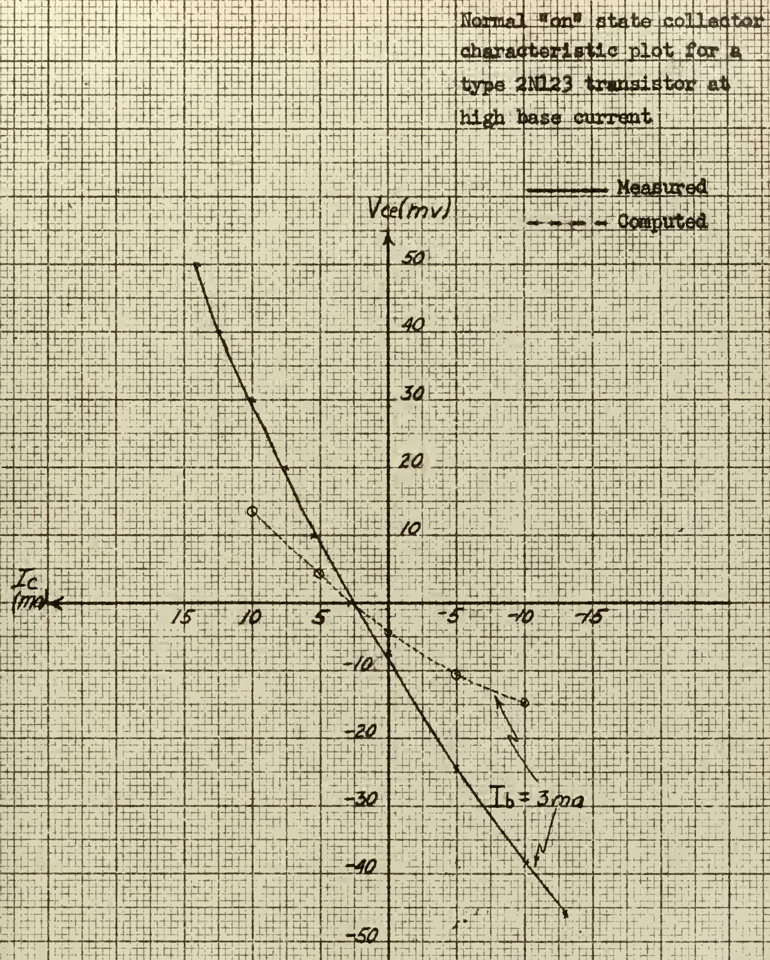

Normal "on" state collector characteristic plot for a type 2N123 transistor at high base current
Measured
Computed
Vce(mv)
50
40
30
20
10
-10
-20
-30
-40
-50
Ic (ma)
15
10
5
-5
-10
-15
Ib = 3ma

Figure (23)

Inverted "on" state collector characteristic plot for a type 2N123 transistor at various base currents

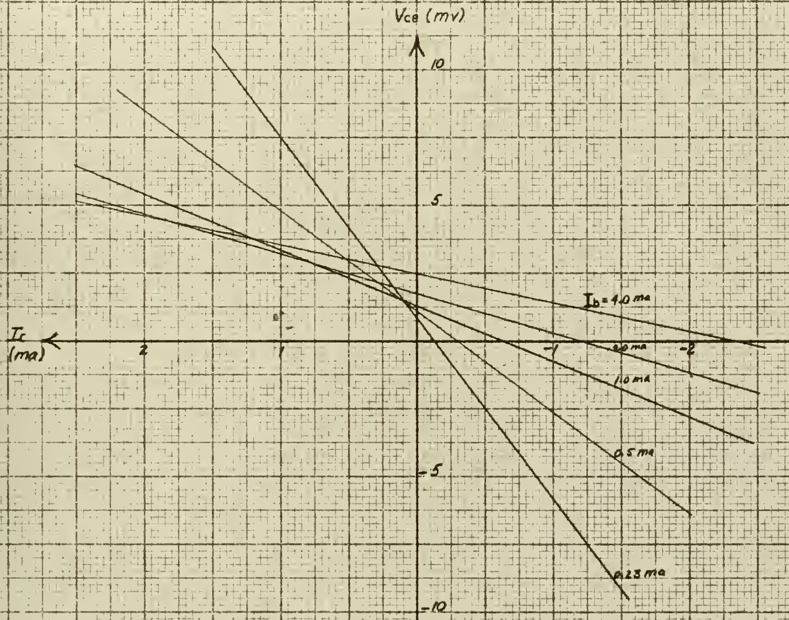
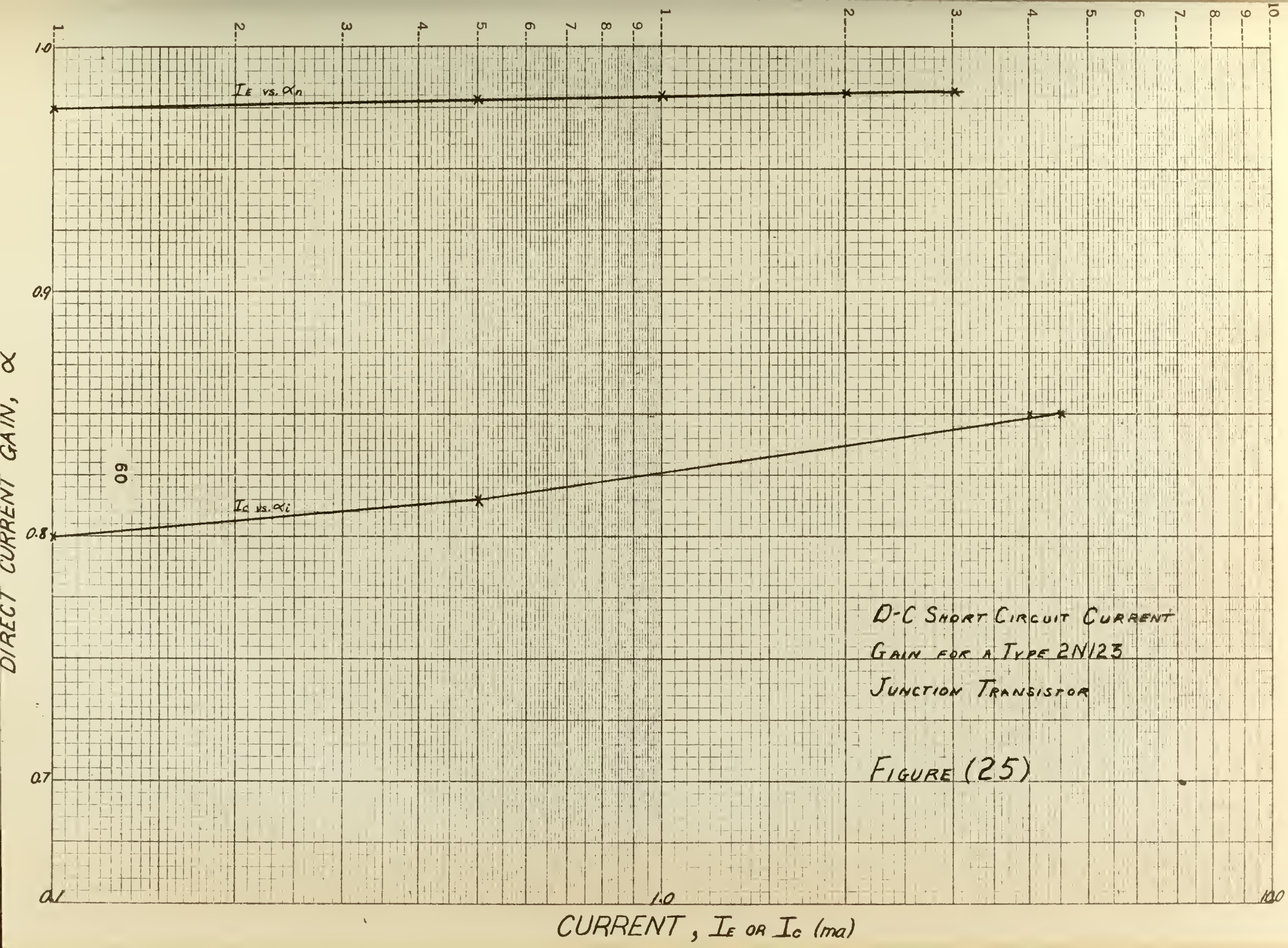


Figure (24)



D-C Short Circuit Current Gain for a Type 2N123 Junction Transistor

Figure (25)

I_e	I_c	I_b	I_c/I_b	α_m	$1-\alpha_m$	$\frac{1-\alpha_m}{\alpha_m}$	$\frac{I_c}{I_b}\frac{(1-\alpha_m)}{\alpha_m}$	$1-\frac{I_c}{I_b}\frac{(1-\alpha_m)}{\alpha_m}$	α_i	$1-\alpha_i$	$\frac{I_c}{I_b}(1-\alpha_i)$	$1+\frac{I_c}{I_b}(1-\alpha_i)$	$\frac{1-\frac{I_c}{I_b}\frac{(1-\alpha_m)}{\alpha_m}}{1+\frac{I_c}{I_b}(1-\alpha_i)}$	$\alpha_i\left[\frac{1-\frac{I_c}{I_b}\frac{(1-\alpha_m)}{\alpha_m}}{1+\frac{I_c}{I_b}(1-\alpha_i)}\right]$	$\ln\alpha_i\left[\frac{1-\frac{I_c}{I_b}\frac{(1-\alpha_m)}{\alpha_m}}{1+\frac{I_c}{I_b}(1-\alpha_i)}\right]$	$V_{ce} = .026\ln\alpha_i\left[\frac{1-\frac{I_c}{I_b}\frac{(1-\alpha_m)}{\alpha_m}}{1+\frac{I_c}{I_b}(1-\alpha_i)}\right]$
+200μa	0	-200μa	0					1.0	.806			1.0	1.0	.806	-.216	-5.6mv
+700	-500μa	-200	2.5	.979	.021	.0214	.046	.954	.815	.185	.463	1.463	.653	.533	-.63	-16.4
+1200	-1000	-200	5.0	.979	.021	.0214	.1072	.893	.826	.134	.67	1.67	.535	.442	-.819	-21.3
-300	+500	-200	-2.5	.977	.023	.0236	-.0579	1.0579	.815	.185	-.463	.537	1.96	1.593	+.466	+12.1
-700	+900	-200	-4.5	.978	.022	.0225	-.101	1.101	.824	.176	-.792	.208	5.29	4.36	1.74	+37.3
+3ma	0	-3ma	0					1	.844			1.0	1.0	.844	-.17	-4.42
+8	-5ma	-3	1.67	.984	.016	.01625	.0251	.9749	.852	.148	.247	1.247	.782	.665	-.408	-10.6
+13	-10	-3	3.33	.985	.015	.0152	.0506	.949	.864	.136	.453	1.453	.653	.564	-.572	-14.8
-2	+5	-3	-1.67	.976	.024	.0246	-.0411	1.0411	.852	.148	-.247	.753	1.382	1.178	.164	4.26
-7	10	-3	-3.33	.978	.022	.0225	-.075	1.075	.864	.136	-.453	.547	1.967	1.70	.532	13.8

Tabular Solutions of Equation (1-12): $V_{ce} = \pm \frac{kT}{q} \ln \left\{ \frac{\alpha_i \left[1 - \frac{I_c}{I_b} \frac{(1-\alpha_m)}{\alpha_m} \right]}{1 + \frac{I_c}{I_b} (1-\alpha_i)} \right\}$

Figure (26)

These calculations are shown in detail in Appendix VI.

Curves for a General Electric type 2N123 p-n-p transistor are included at this point to show their similarity to the curves for the n-p-n unit just described. Figs. (22) and (23) show the "on" state plot for values of $I_b = 0.2$ ma and $I_b = 3$ ma respectively. Calculated and measured curves in Fig. (22) agree very well because current levels are low and voltage drops across R_{es} and R_{cs} are small. The curves of Fig.(23) show the disagreement between measured and calculated curves which may be expected when currents are increased. Fig. (24) is of interest because it shows the inverted "on" state curves of the transistor whose normal "on" state curves are plotted in Figs. (22) and (23). Fig. (24) shows that $V_{ec}\big|_{I_e=0}$ is less than $V_{ce}\big|_{I_c=0}$ and, therefore, that this transistor will be a better switch in its "on" state if it is operated inverted than if it is operated normally. Data and calculations for these curves are shown in Figs. (24), (25) and (26). A comparison of the curves for this p-n-p unit with those of the preceding n-p-n unit (Fig. 16) shows very little difference between the two except that the base drive currents and the voltage offsets are of opposite polarity. In fact, all of the curves which were obtained for good switching transistors were very much like those which have just been discussed.

The conclusions which are to be drawn from the comparisons and calculations which have been presented are:

1. The equations in Chapter I may be used without correction only at low voltages and currents.

2. With corrections for lead resistances and leakage resistances, these same equations will give good agreement at moderate voltages and currents. Obtaining these corrections has, however, involved measuring

the actual collector characteristics for at least one value of I_b.

3. At high collector voltages, breakdown effects become prominent and the equations of Chapter II no longer apply.

4. In view of the limited range of application for the equations of Chapter I, the difficulty of obtaining the lead and leakage resistances and, finally, the failure of these equations near voltage breakdown of the transistor, the device described in Chapter III appears to fill a need for direct measurement of the static switching properties of junction transistors.

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This appendix shows the derivation of the diffusion equation (1-2) and its solution to obtain the equations relating collector current and emitter current to the voltages applied to the collector and emitter junctions of a p-n-p transistor.

The diffusion equation for holes in a slab of n-type germanium is given in Chapter I as,

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} + \frac{p_n - p}{\tau_p} \qquad (1\text{-}2)$$

where,

p = the concentration of holes at any time, t, and at any point, x, in the slab.

t = time.

x = distance into the slab measured along the normal to its surface.

p_n = the thermal equilibrium concentration of holes in the n-type germanium.

p = the "hole lifetime." It is the time required for a group of holes injected into the slab to be reduced in number by a factor of $1/e$. (e is the natural log base.)

D_p = the diffusion constant for holes in the slab of n-type germanium. It has the dimensions length ^{2}/time.

Since the base region of a transistor is just such a slab of semiconductor, the solution of this equation for the appropriate boundary conditions should lead to expressions relating the flow of current across the emitter and the collector junctions with the voltages across these junctions. Ebers and Moll (5) have worked out a solution in three dimensions with very few restrictions on geometry; however, the problem is simplified and made easier to understand if it is limited to one dimension.

This can be done by considering the junctions to be parallel planes separated by an infinite slab, the base. The problem is solved for this geometry in Chapter 8 of TRANSISTOR ELECTRONICS, (7).

The assumptions upon which the derivations of the equations to follow is based are:

1. The resistivity of all semiconductor regions is low enough that no voltage drops occur except across the p-n junctions.

2. The emitter efficiency is not a function of emitter current.

3. Space-charge layer widening effects are negligible (4).

4. Emitter and collector junctions separately, have voltage-current relationships of the form,

$$I = I_S \left(e^{qV/kT} - 1\right) \qquad (1\text{-}1)$$

where,

I = the junction current.

I_s = the reverse saturation current across the junction due to the flow of minority carriers.

q = electronic charge = 1.6×10^{-19} coulombs.

k = Boltzmann's constant = 1.38×10^{-23} Joules/°K molecule.

T = temperature in degrees Kelvin.

5. The emitter, base and collector regions, each separately, consist of homogeneous n or p type semiconductor.

Equation (1-2) is derived from a law governing the diffusion process which states:

> The rate of diffusion of a substance is proportional to the cross-sectional area through which it moves and to the concentration gradient of the substance (7).

For the germanium slab, the transistor base in this problem, the law can be written,

$$I_p(x) = \frac{\partial Q_p}{\partial t} = -q D_p \frac{\partial p}{\partial x} \qquad \text{(A1-1)}$$

where, $I_p(x)$ = That part of the current flow in the x-direction at any point, x, in the base due to the movement of holes.

Q_p = the total charge of all holes in a volume of the base of unit cross-section and length, W, where the unit cross-section is taken in a plane parallel to the base surface and the length, W, the base thickness, is taken along the x-direction.

Differentiating (A1-1) with respect to x yields,

$$\frac{\partial^2 Q_p}{\partial t \partial x} = -q D_p \frac{\partial^2 p}{\partial x^2} \qquad \text{(A1-2)}$$

Differentiating the expression defining p,

$$p = -\frac{1}{q} \frac{\partial Q_p}{\partial x} \qquad \text{(A1-3)}$$

with respect to t gives,

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial^2 Q_p}{\partial x \partial t} \qquad \text{(A1-4)}$$

If (A1-2) and (A1-4) are combined to eliminate Q_p, the result is,

$$-q \frac{\partial p}{\partial t} = -q D_p \frac{\partial^2 p}{\partial x^2}$$

or,

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} \qquad \text{(A1-5)}$$

The time rate decay of a group of holes injected into a piece of n-type germanium is noted from experiment to be an exponential function (7). The time required for the number of holes in the group to decay to a factor of 1/e is called τ_p. The concentration of holes at any instant

in the n-material after an initial increase in concentration, p_1' has been made is,

$$p = p_n + p_1' e^{-t/\tau_p} \tag{A1-6}$$

if this equation is differentiated with respect to time, the result is,

$$\frac{\partial p}{\partial t} = -\frac{1}{\tau_p} p_1' e^{-t/\tau_p} \tag{A1-7}$$

combining (A1-6) and (A1-7) to eliminate the exponential factor gives,

$$p = p_n - \tau_p \, \partial p/\partial t$$

or,

$$\frac{\partial p}{\partial t} = \frac{p_n - p}{\tau_p} \tag{A1-8}$$

This is the decay rate of hole concentration due to recombination of holes and electrons in the base. When equations (A1-5) and (A1-8) are added together, the result is,

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} + \frac{p_n - p}{\tau_p} \tag{1-2}$$

Assume that a general solution to (1-2) is,

$$p = P_0(x) + p_1(x) e^{j\omega t} \tag{A1-9}$$

where,

$P_0(x)$ = the static component of hole concentration.

$p_1(x)$ = the time varying component of hole concentration at any point, x, in the base.

differentiating (A1-9), one obtains,

$$\frac{\partial p}{\partial t} = j\omega\, p_1(x)\, e^{j\omega t}$$

$$\frac{\partial p}{\partial x} = \frac{\partial P_0(x)}{\partial x} + \frac{\partial p_1(x)}{\partial x}\, e^{j\omega t}$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 P_0(x)}{\partial x^2} + \frac{\partial^2 p_1(x)}{\partial x^2}\, e^{j\omega t}$$

Substituting (A1-9) and its derivatives into (1-2) yields,

$$j\omega\, p_1(x) e^{j\omega t} = D_p\left[\frac{\partial^2 P_0(x)}{\partial x^2} + \frac{\partial^2 p_1(x)}{\partial x}\, e^{j\omega t}\right] + \frac{p_n - \left[P_0(x) + p_1(x) e^{j\omega t}\right]}{\tau_p} \qquad \text{(A1-10)}$$

This equation can be broken into a-c and d-c components. Since this paper is concerned only with the d-c behavior, the a-c portion of this equation will be discarded. The d-c equation, then is,

$$D_p\, \frac{\partial^2 P_0(x)}{\partial x} = \frac{p_n - P_0(x)}{\tau_p} \qquad \text{(A1-11)}$$

To remove time completely from (A1-11), the substitution,

$$\tau_p = \frac{L_p^2}{D_p} \qquad \text{(A1-12)}$$

is made. L_p is the diffusion length for holes in the base region; this is the distance measured in the + x-direction in which the hole concentration decreases by a factor of 1/e, assuming that there are no hole sources or electric fields along this length. When this substitution is made, (A1-11) becomes,

$$\frac{\partial^2 P_0(x)}{\partial x} = \frac{p_n - P_0(x)}{L_p^2} \qquad \text{(A1-13)}$$

An equation similar to (2-1) can be written to describe the hole concentration on the n-side of a p-n junction when a voltage is applied to the junction,

$$P_o(0) = p_n e^{\frac{q\varphi_e}{kT}} \qquad \text{(A1-14)}$$

where,

k = Boltzmann's constant = 1.38 x 10^{-23} Joules/°K molecule.

T = temperature in degrees Kelvin.

If one of the surfaces of the base forms an emitter junction, then (A1-14) gives the hole concentration at this base surface in terms of the voltage across the junction. Let the voltage, φ_e, be the voltage of the p-material (the emitter) measured with respect to the n-material (the base).

Another equation can be written for hole concentration at the opposite side of the base where x = W, the width of the base:

$$P_o(W) = p_n e^{\frac{q\varphi_c}{kT}} \qquad \text{(A1-15)}$$

Let this side of the base form the collector junction and let φ_c be the voltage of the p-material (the collector) measured with respect to the n-material (the base). Equations (A1-14) and (A1-15) are the boundary conditions which will be applied to (A1-13).

Assume that the general solution to (A1-13) is,

$$P_o(x) = A \sinh \frac{x}{L_p} + B \cosh \frac{x}{L_p} + p_n \qquad \text{(A1-16)}$$

where A and B are constants to be determined by applying the boundary conditions. Differentiating (A1-16), one obtains,

$$\frac{\partial P_0(x)}{\partial x} = \frac{A}{L_p} \cosh \frac{x}{L_p} + \frac{B}{L_p} \sinh \frac{x}{L_p}$$

$$\frac{\partial^2 P_0(x)}{\partial x^2} = \frac{A}{L_p^2} \sinh \frac{x}{L_p} + \frac{B}{L_p^2} \cosh \frac{x}{L_p}$$

Substituting (A1-16) and its derivatives into (A1-13) gives the result,

$$A/L_p^2 \quad \sinh \frac{x}{L_p} + \frac{B}{L_p^2} \cosh \frac{x}{L_p} = \frac{\left(A \sinh \frac{x}{L_p} + B \cosh \frac{x}{L_p} + p_n\right) - p_n}{L_p^2}$$

which verifies the solution.

To evaluate the constant B, equation (A1-14) is substituted into (A1-16); this yields, $p_n e^{\frac{q\varphi_e}{kT}} = A \sinh 0 + B \cosh 0 + p_n$

$$B = p_n \left(e^{\frac{q\varphi_e}{kT}} - 1\right) \qquad \text{(A1-17)}$$

To evaluate the constant A, Equations (A1-15) and (A1-17) are substituted into (A1-16) giving,

$$p_n e^{\frac{q\varphi_c}{kT}} = A \sinh \frac{W}{L_p} - p_n \left(1 - e^{\frac{q\varphi_e}{kT}}\right) \cosh \frac{W}{L_p} + p_n$$

solving this for A,

$$A = -\frac{1}{\sinh \frac{W}{L_p}} \left[-p_n \left(1 - e^{\frac{q\varphi_e}{kT}}\right) \cosh \frac{W}{L_p} + p_n \left(1 - e^{\frac{q\varphi_c}{kT}}\right) \right]$$

or,

$$A = p_n \left[\coth \frac{W}{L_p} \left(1 - e^{\frac{q\varphi_e}{kT}}\right) - \operatorname{csch} \frac{W}{L_p} \left(1 - e^{\frac{q\varphi_c}{kT}}\right) \right] \qquad \text{(A1-18)}$$

When (A1-18) is substituted into (A1-16), the d-c solution for the hole concentration at any point, x, in the base is obtained,

$$P_0(x) = p_n\left[\left(1-e^{\frac{q\psi_e}{kT}}\right)\coth\frac{W}{L_p} - \left(1-e^{\frac{q\psi_c}{kT}}\right)\operatorname{csch}\frac{W}{L_p}\right]\sinh\frac{x}{L_p}$$

$$- p_n\left(1-e^{\frac{q\psi_e}{kT}}\right)\cosh\frac{x}{L_p} + p_n \qquad \text{(A1-19)}$$

The d-c current, $I_p(x)$, at any point, x, in the base may be found by differentiating (A1-19) with respect to x and substituting the result into (A1-1) as follows:

$$\frac{\partial P_0(x)}{\partial x} = \frac{p_n}{L_p}\left[\left(1-e^{\frac{q\psi_e}{kT}}\right)\coth\frac{W}{L_p} - \left(1-e^{\frac{q\psi_c}{kT}}\right)\operatorname{csch}\frac{W}{L_p}\right]\cosh\frac{x}{L_p}$$

$$- \frac{p_n}{L_p}\left(1-e^{\frac{q\psi_e}{kT}}\right)\sinh\frac{x}{L_p} \qquad \text{(A1-20)}$$

$$I_p(x) = \frac{-qD_p p_n}{L_p}\left[\left(1-e^{\frac{q\psi_e}{kT}}\right)\coth\frac{W}{L_p} - \left(1-e^{\frac{q\psi_c}{kT}}\right)\operatorname{csch}\frac{W}{L_p}\right]\cosh\frac{x}{L_p}$$

$$+ \frac{qD_p p_n}{L_p}\left(1-e^{\frac{q\psi_e}{kT}}\right)\sinh\frac{x}{L_p} \qquad \text{(A1-21)}$$

Hole currents at the emitter and collector junctions may be obtained from (A1-21) by the substitutions, x = 0 to get the emitter hole current and x = W to get the collector hole current. This is done as follows:

At the emitter,

$$I_p(0) \equiv I_{e_p} = -\frac{qD_p p_n}{L_p}\left[\left(1-e^{\frac{q\psi_e}{kT}}\right)\coth\frac{W}{L_p} \right. \qquad \text{(A1-22)}$$

$$\left. - \left(1-e^{q\psi_c/kT}\right)\operatorname{csch}\frac{W}{L_p}\right]$$

At the collector,

$$I_p(W) \equiv -I_{c_p} = -\frac{qD_p p_n}{L_p}\left[\left(1-e^{\frac{q\psi_e}{kT}}\right)\coth\frac{W}{L_p} - \left(1-e^{\frac{q\psi_c}{kT}}\right)\operatorname{csch}\frac{W}{L_p}\right]\cosh\frac{W}{L_p}$$

$$+ \frac{qD_p p_n}{L_p}\left(1-e^{\frac{q\psi_e}{kT}}\right)\sinh\frac{W}{L_p}$$

simplifying,

$$I_{c_p} = \frac{q D_p p_n}{L_p}\left[\left(1-e^{\frac{q\varphi_e}{kT}}\right)\frac{\cosh^2\frac{W}{L_p}}{\sinh\frac{W}{L_p}} - \left(1-e^{\frac{q\varphi_e}{kT}}\right)\frac{\sinh^2\frac{W}{L_p}}{\sinh\frac{W}{L_p}} - \left(1-e^{\frac{q\varphi_c}{kT}}\right)\frac{\cosh\frac{W}{L_p}}{\sinh\frac{W}{L_p}}\right]$$

$$I_{c_p} = \frac{q D_p p_n}{L_p}\left[\left(1-e^{\frac{q\varphi_e}{kT}}\right)\operatorname{csch}\frac{W}{L_p} - \left(1-e^{\frac{q\varphi_c}{kT}}\right)\coth\frac{W}{L_p}\right] \qquad \text{(A1-23)}$$

Nothing has been said about electron current in the preceding derivations, although it is obvious that electrons must flow from the base to the emitter when holes flow from emitter to base. To get the total current flow across the emitter and collector junctions, the electron current must be added to the hole current. Fortunately, to find this current another solution of the diffusion equation is not necessary. It happens that the solution for the electron current components of I_e and I_c is similar enough to the solution for the hole current components that the former may be found simply from an inspection of the hole current solution.

If it is assumed that the distance from the emitter junction to the ohmic contact of the supply lead to the emitter region is much greater than L_n, the diffusion length for electrons (analogous to L_p for holes), then all the electrons emitted from the base will have recombined with holes before they have reached the emitter region ohmic contact. For practical purposes, the width of the emitter region may be considered to be infinite and the equation (A1-22) may be written for the electron component of emitter current, I_{e_n}, as,

$$I_{e_n} = \frac{-q D_n n_p}{L_n}\left(1-e^{\frac{q\varphi_e}{kT}}\right) \qquad \text{(A1-24)}$$

where,

n_p = the thermal equilibrium concentration of electrons in the emitter region.

D_n = the diffusion constant for electrons in the p-regions.

L_n = the diffusion length for electrons in the emitter region.

A similar line of reasoning applied to the collector junction permits the electron component of collector current, I_{c_n} to be written by inspection of equation (A1-23),

$$I_{c_n} = -\frac{q D_n n_p'}{L_n'}\left(1 - e^{\frac{q\varphi_c}{kT}}\right) \qquad \text{(A1-25)}$$

where,

n_p' = the thermal equilibrium concentration of electrons in the collector region.

L_n' = the diffusion constant for electrons in the collector region.

The total emitter current is given by the equation,

$$I_e = I_{e_p} + I_{e_n} \qquad \text{(A1-26)}$$

Substituting (A1-22) and (A1-24) into (A1-26) gives,

$$I_e = \frac{-q D_p p_n}{L_p}\left[\left(1 - e^{\frac{q\varphi_e}{kT}}\right)\coth\frac{W}{L_p} - \left(1 - e^{\frac{q\varphi_c}{kT}}\right)\operatorname{csch}\frac{W}{L_p}\right] - \frac{q D_n n_p}{L_n}\left(1 - e^{\frac{q\varphi_e}{kT}}\right)$$

simplifying,

$$I_e = -\left(\frac{q D_p p_n}{L_p}\coth\frac{W}{L_p} + \frac{q D_n n_p}{L_n}\right)\left(1 - e^{\frac{q\varphi_e}{kT}}\right) + \frac{q D_p p_n}{L_p}\left(1 - e^{\frac{q\varphi_c}{kT}}\right)\operatorname{csch}\frac{W}{L_p}$$

$$I_e = \frac{q D_p p_n}{L_p}\left[\left(\coth\frac{W}{L_p} + \frac{D_n n_p L_p}{D_p p_n L_n}\right)\left(e^{\frac{q\varphi_e}{kT}} - 1\right) - \operatorname{csch}\frac{W}{L_p}\left(e^{\frac{q\varphi_c}{kT}} - 1\right)\right] \qquad \text{(A1-27)}$$

This equation may be written more simply, as shown in Chapter I, as,

$$I_e = a_{11}\left(e^{\frac{q\varphi_e}{kT}}-1\right) + a_{12}\left(e^{\frac{q\varphi_c}{kT}}-1\right)$$

where, $$a_{11} = \frac{qD_p p_n}{L_p}\left[\coth\frac{W}{L_p} + \frac{D_n n_p L_p}{D_p p_n L_n}\right]$$

$$a_{12} = -\frac{qD_p p_n}{L_p}\operatorname{csch}\frac{W}{L_p}$$

Substituting (A1-23) and (A1-25) into the equation,

$$I_c = I_{c_p} + I_{c_n} \tag{A1-28}$$

gives,

$$I_c = \frac{qD_p p_n}{L_p}\left[\left(1-e^{\frac{q\varphi_e}{kT}}\right)\operatorname{csch}\frac{W}{L_p} - \left(1-e^{\frac{q\varphi_c}{kT}}\right)\coth\frac{W}{L_p}\right]$$

$$- \frac{qD_n n_p'}{L_n'}\left(1-e^{\frac{q\varphi_c}{kT}}\right)$$

simplifying,

$$I_c = -\left[\frac{qD_p p_n}{L_p}\coth\frac{W}{L_p} + \frac{qD_n n_p'}{L_n'}\right]\left(1-e^{\frac{q\varphi_c}{kT}}\right) + \frac{qD_p p_n}{L_p}\left(1-e^{\frac{q\varphi_e}{kT}}\right)\operatorname{csch}\frac{W}{L_p}$$

$$I_c = \frac{qD_p p_n}{L_p}\left[\left(\coth\frac{W}{L_p} + \frac{D_n n_p' L_p}{D_p p_n L_n'}\right)\left(e^{\frac{q\varphi_c}{kT}}-1\right) - \operatorname{csch}\frac{W}{L_p}\left(e^{\frac{q\varphi_e}{kT}}-1\right)\right] \tag{A1-29}$$

This equation may be written as shown in Chapter I:

$$I_c = a_{21}\left(e^{\frac{q\varphi_e}{kT}}-1\right) + a_{22}\left(e^{\frac{q\varphi_c}{kT}}-1\right) \tag{1-4}$$

where,

$$a_{21} = a_{12} = \frac{-q D_p p_n}{L_p} \operatorname{csch} \frac{W}{L_p}$$

$$a_{22} = \frac{q D_p p_n}{L_p} \left[\coth \frac{W}{L_p} + \frac{D_n n_p' L_p}{D_p p_n L_n'} \right]$$

APPENDIX II

This appendix shows the derivations in terms of small signal parameters of the coefficients, a_{11}, a_{12}, a_{21} and a_{22} of the general equations (1-3) and (1-4) relating the transistor terminal voltages and currents. The derivations are as presented by Ebers and Moll (5).

For normal operation in Regions I and II, equations (1-3) and (1-4) may be simplified to,

$$I_e = a_{11}\left(e^{\frac{q\varphi_e}{kT}}-1\right) - a_{12} \tag{A2-1}$$

$$I_c = a_{21}\left(e^{\frac{q\varphi_e}{kT}}-1\right) - a_{22} \tag{A2-2}$$

because in these regions, $e^{\frac{q\varphi_c}{kT}} << 1$

For inverted operation in Regions I and II, equations (1-3) and (1-4) may be written,

$$I_e = -a_{11} + a_{12}\left(e^{\frac{q\varphi_c}{kT}}-1\right) \tag{A2-3}$$

$$I_c = -a_{21} + a_{22}\left(e^{\frac{q\varphi_c}{kT}}-1\right) \tag{A2-4}$$

where, now, $e^{\frac{q\varphi_e}{kT}} << 1$

When (A2-1) and (A2-2) are combined to eliminate the exponential factor, the result is,

$$I_c = \frac{a_{21}}{a_{11}} I_e + \left(\frac{a_{12}\, a_{21}}{a_{11}} - a_{22}\right) \tag{A2-5}$$

Similarily, when (A2-3) and (A2-4) are combined to eliminate the exponential factor, the result is,

$$I_e = \frac{a_{12}}{a_{22}} I_c + \left(\frac{a_{12}\, a_{21}}{a_{22}} - a_{11} \right) \quad \text{(A2-6)}$$

Equations (A2-5) and (A2-6) can also be written in terms of small signal parameters as,

$$I_c = -\alpha_n I_e + I_{co} \quad \text{(A2-7)}$$

$$I_e = -\alpha_i I_c + I_{eo} \quad \text{(A2-8)}$$

where,

α_n = the normal d-c current gain of the transistor defined by the equation, $\alpha_n = \left. \frac{I_c}{I_e} \right|_{V_{cb}=0}$

α_i = the inverted d-c current gain of the transistor defined by the equation, $\alpha_i = \left. \frac{I_e}{I_c} \right|_{V_{eb}=0}$

V_{cb} = the voltage applied to the collector region measured with respect to the base region.

V_{eb} = the voltage applied to the emitter region measured with respect to the base region.

I_{co} = the reverse saturation current of the emitter junction.

I_{eo} = The reverse saturation current of the collector junction.

Equating (A2-5) with (A2-7), and (A2-6) with (A2-8) term for term yields the following:

$$-\alpha_n = \frac{a_{21}}{a_{11}} \quad \text{(A2-9)}$$

$$-\alpha_i = \frac{a_{12}}{a_{22}} \quad \text{(A2-10)}$$

$$I_{co} = \left(\frac{a_{12}\, a_{21}}{a_{11}} - a_{22}\right) \tag{A2-11}$$

$$I_{eo} = \left(\frac{a_{12}\, a_{21}}{a_{22}} - a_{11}\right) \tag{A2-12}$$

These equations are solved for the "a" coefficients as follows:

Substitute (A2-9) and (A2-10) into (A2-11),

$$I_{co} = \left(-\alpha_m\, a_{12} + \frac{a_{12}}{\alpha_i}\right)$$

and solve for a_{12}.

$$a_{12} = \frac{\alpha_i\, I_{co}}{1 - \alpha_m \alpha_i} \tag{1-6}$$

Substitute (A2-9) and (A2-10) into (A2-12)

$$I_{eo} = \left(-\alpha_i\, a_{21} + \frac{a_{21}}{\alpha_m}\right)$$

and solve for a_{21}.

$$a_{21} = \frac{\alpha_m\, I_{eo}}{1 - \alpha_m \alpha_i} \tag{1-7}$$

To find a_{11}, substitute (1-7) into (A2-9).

$$a_{11} = \frac{-\, I_{eo}}{1 - \alpha_m \alpha_i} \tag{1-5}$$

To find a_{22}, substitute (1-6) into (A2-10).

$$a_{22} = \frac{-\, I_{co}}{1 - \alpha_m \alpha_i} \tag{1-8}$$

This appendix derives the equations for V_{ce} and V_{ec} in terms of measurable small signal parameters. The derivations are as presented by Ebers and Moll (5).

Equations (1-10) and (1-11) may be solved for the exponential factors of I_e and I_c as follows:

$$I_e = \frac{-I_{eo}}{1-\alpha_n\alpha_i}\left(e^{\frac{q\varphi_e}{kT}}-1\right) + \frac{\alpha_i I_{co}}{1-\alpha_n\alpha_i}\left(e^{\frac{q\varphi_c}{kT}}-1\right) \tag{1-10}$$

$$I_c = \frac{\alpha_n I_{eo}}{1-\alpha_n\alpha_i}\left(e^{\frac{q\varphi_e}{kT}}-1\right) - \frac{I_{co}}{1-\alpha_n\alpha_i}\left(e^{\frac{q\varphi_c}{kT}}-1\right) \tag{1-11}$$

Using Cramer's Rule,

$$\left(e^{\frac{q\varphi_e}{kT}}-1\right) = \frac{\begin{vmatrix} I_e & \dfrac{\alpha_i I_{co}}{1-\alpha_n\alpha_i} \\ I_c & \dfrac{-I_{co}}{1-\alpha_n\alpha_i} \end{vmatrix}}{\begin{vmatrix} \dfrac{-I_{eo}}{1-\alpha_n\alpha_i} & \dfrac{\alpha_i I_{co}}{1-\alpha_n\alpha_i} \\ \dfrac{\alpha_n I_{eo}}{1-\alpha_n\alpha_i} & \dfrac{-I_{co}}{1-\alpha_n\alpha_i} \end{vmatrix}}$$

Simplifying the determinant gives,

$$\left(e^{\frac{q\varphi_e}{kT}}-1\right) = \frac{\dfrac{I_{co}}{1-\alpha_n\alpha_i}\begin{vmatrix} I_e & \alpha_i \\ I_c & -1 \end{vmatrix}}{\dfrac{I_{eo}I_{co}}{(1-\alpha_n\alpha_i)^2}\begin{vmatrix} -1 & \alpha_i \\ \alpha_n & -1 \end{vmatrix}}$$

Expanding and cancelling common terms produces,

$$\left(e^{\frac{q\varphi_e}{kT}}-1\right) = \frac{-I_e - \alpha_i I_c}{\frac{I_{eo}}{1-\alpha_n\alpha_i}\left(1-\alpha_n\alpha_i\right)}$$

$$\left(e^{\frac{q\varphi_e}{kT}}-1\right) = -\frac{1}{I_{eo}}\left(I_e+\alpha_i I_c\right) \tag{A3-1}$$

To find the other exponential terms, Cramer's Rule is used again,

$$\left(e^{\frac{q\varphi_c}{kT}}-1\right) = \frac{\begin{vmatrix} \frac{-I_{eo}}{1-\alpha_n\alpha_i} & I_e \\ \frac{\alpha_n I_{eo}}{1-\alpha_n\alpha_i} & I_c \end{vmatrix}}{\begin{vmatrix} \frac{-I_{eo}}{1-\alpha_n\alpha_i} & \frac{\alpha_i I_{co}}{1-\alpha_n\alpha_i} \\ \frac{\alpha_n I_{eo}}{1-\alpha_n\alpha_i} & \frac{-I_{co}}{1-\alpha_n\alpha_i} \end{vmatrix}}$$

simplifying the determinant gives,

$$\left(e^{\frac{q\varphi_c}{kT}}-1\right) = \frac{\frac{I_{eo}}{1-\alpha_n\alpha_i}\begin{vmatrix} -1 & I_e \\ \alpha_n & I_c \end{vmatrix}}{\frac{I_{eo}I_{co}}{(1-\alpha_n\alpha_i)^2}\begin{vmatrix} -1 & \alpha_i \\ \alpha_n & -1 \end{vmatrix}}$$

which reduces to,

$$\left(e^{\frac{q\varphi_c}{kT}}-1\right) = -\frac{1}{I_{co}}\left(I_c+\alpha_n I_e\right) \tag{A3-2}$$

V_{ce} is defined as the collector voltage measured with respect to the emitter; therefore,

$$V_{ce} = \pm\left(\varphi_c - \varphi_e\right) \tag{A3-3}$$

where the (+) sign is used with p-n-p transistors and the (-) sign is used with n-p-n transistors.

When equations (A3-1) and (A3-2) are solved, respectively for φ_e and φ_c, the result is,

$$\varphi_e = \frac{kT}{q} \ln \left[1 - \frac{I_e + \alpha_i I_c}{I_{eo}} \right] \tag{A3-4}$$

$$\varphi_c = \frac{kT}{q} \ln \left[1 - \frac{I_c + \alpha_m I_e}{I_{co}} \right] \tag{A3-5}$$

Substituting (A3-4) and (A3-5) into (A3-3) yields,

$$V_{ce} = \pm \frac{kT}{q} \ln \frac{\dfrac{I_c + \alpha_m I_e}{I_{co}} + 1}{\dfrac{I_e + \alpha_i I_c}{I_{eo}} + 1}$$

It is found from Kirchoff's current law that $I_e + I_b + I_c = 0$ and that substituting $-(I_b + I_c)$ for I_b gives,

$$V_{ce} = \pm \frac{kT}{q} \frac{\dfrac{I_c - \alpha_m (I_b + I_c)}{I_{co}} + 1}{\dfrac{-(I_b + I_c) + \alpha_i I_c}{I_{eo}} + 1}$$

Simplifying this expression produces,

$$V_{ce} = \pm \frac{kT}{q} \ln \left[\frac{I_c - \alpha_m I_b - \alpha_m I_c + I_{co}}{-I_b - I_c + \alpha_i I_c + I_{eo}} \cdot \frac{I_{eo}}{I_{co}} \right]$$

It is noted that, $\alpha_m I_{eo} = \alpha_i I_{co}$ (1-9)

replacing I_{eo} with $\dfrac{\alpha_i}{\alpha_m} I_{co}$ yields,

$$V_{ce} = \pm \frac{kT}{q} \ln \left[\frac{I_c(1-\alpha_n) - \alpha_n I_b + I_{co}}{-I_b - I_c(1-\alpha_i) + \frac{\alpha_i}{\alpha_n} I_{co}} \cdot \frac{\alpha_i}{\alpha_n} I_{co} \cdot \frac{1}{I_{co}} \right]$$

Simplifying again,

$$V_{ce} = \pm \frac{kT}{q} \ln \left\{ \frac{\alpha_i \left[\frac{I_c}{I_b} \frac{(1-\alpha_n)}{\alpha_n} - \frac{I_b}{I_b} + \frac{I_{co}}{I_b \alpha_n} \right]}{-\left[1 + \frac{I_c}{I_b}(1-\alpha_i) \right] + \frac{\alpha_i}{\alpha_n} \frac{I_{co}}{I_b}} \right\}$$

$$V_{ce} = \pm \frac{kT}{q} \ln \left\{ \frac{\alpha_i \left[1 - \frac{I_c}{I_b} \frac{(1-\alpha_n)}{\alpha_n} \right] - \frac{\alpha_i}{\alpha_n} \frac{I_{co}}{I_b}}{\left[1 + \frac{I_c}{I_b}(1-\alpha_i) \right] + \frac{\alpha_i}{\alpha_n} \frac{I_{co}}{I_b}} \right\} \quad \text{(A3-6)}$$

The factor $\frac{\alpha_i}{\alpha_n} \frac{I_{co}}{I_b}$ is negligibly small and may be dropped; V_{ce} is then,

$$V_{ce} = \pm \frac{kT}{q} \ln \left\{ \frac{\alpha_i \left[1 - \frac{I_c}{I_b} \cdot \frac{(1-\alpha_n)}{\alpha_n} \right]}{1 + \frac{I_c}{I_b} (1-\alpha_i)} \right\} \quad \text{(1-12)}$$

To obtain the equation for V_{ec} the subscripts "n" and "i," and "c" and "e" are interchanged in equation (1-12)

$$V_{ec} = \pm \frac{kT}{q} \ln \left\{ \frac{\alpha_n \left[1 - \frac{I_e}{I_b} \frac{(1-\alpha_i)}{\alpha_i} \right]}{1 + \frac{I_e}{I_b} (1-\alpha_n)} \right\} \quad \text{(1-13)}$$

This appendix derives the equation for dynamic collector resistance, r_{ce}, in terms of measurable small signal parameters using the method followed by Ebers and Moll (5).

Starting with the equation for V_{ce} derived in Appendix III,

$$V_{ce} = \pm \frac{kT}{q} \ln \left\{ \frac{\alpha_i \left[1 - \frac{I_c}{I_b} \frac{(1-\alpha_n)}{\alpha_n} \right] - \frac{\alpha_i}{\alpha_n} \frac{I_{co}}{I_b}}{\left[1 + \frac{I_c}{I_b}(1-\alpha_i) \right. + \frac{\alpha_i}{\alpha_n} \frac{I_{co}}{I_b}} \right\} \qquad \text{(A3-6)}$$

this is put into simpler mathematical form by writing,

$$V_{ce} = B \ln \left\{ \frac{a(1-bx)-d}{(1+ex)+d} \right\}$$

where,

$a = \alpha_i$

$b = \dfrac{1-\alpha_n}{I_b \alpha_n}$

$d = \dfrac{\alpha_i}{\alpha_n} \cdot \dfrac{I_{co}}{I_b}$

$e = \dfrac{1-\alpha_i}{I_b}$

$B = kT/q$

$V = V_{ce}$

$x = I_c$

rearranging this,

$$V = B \ln \frac{(a-d) - abx}{(1+d) + ex}$$

This can be put into simpler mathematical form as,

$$V = B \ln \frac{\alpha - \beta x}{\gamma - \omega x}$$

where,

$$\alpha = (a-d)$$

$$\beta = ab$$

$$\gamma = (1+d)$$

$$\omega = -e$$

Differentiating this expression for V_{ce} gives,

$$\frac{dV}{dx} = B\,\frac{-\beta(\gamma-\omega x)-\omega(\alpha-\beta x)}{(\gamma-\omega x)^2}\cdot\frac{(\gamma-\omega x)}{(\alpha-\beta x)}$$

substituting the previous set of constants back into the equations gives,

$$\frac{dV}{dx} = B\left[\frac{-ab}{(a-d)-abx} + \frac{e}{(1+d)+ex}\right]$$

and substituting the original set of constants into this equation gives,

$$r_{ce} = \frac{dV_{ce}}{dI_c} = \pm\frac{kT}{q}\left[\frac{-(1-\alpha_n)}{(\alpha_n I_b - I_{co})-(1-\alpha_n)I_c} + \frac{-(1-\alpha_i)}{I_b + \frac{\alpha_i}{\alpha_n}I_{co} + (1-\alpha_i)I_c}\right]$$

simplifying,

$$r_{ce} = \frac{dV_{ce}}{dI_c} = \pm\frac{kT}{q}\left[\frac{-(1-\alpha_n)}{\alpha_n\left[I_b - I_c\frac{(1-\alpha_n)}{\alpha_n}\right]-I_{co}} + \frac{-(1-\alpha_i)}{I_b + I_c(1-\alpha_i) + \frac{\alpha_i}{\alpha_n}I_{co}}\right]$$

(1-14)

The equation for r_{ec} is found by interchanging the subscripts "n" and "i," and "e" and "c," as follows:

$$r_{ec} = \frac{dV_{ec}}{dI_c} = \pm\frac{kT}{q}\left[\frac{-(1-\alpha_n)}{I_b + I_e(1-\alpha_n) + \frac{\alpha_n}{\alpha_i}I_{eo}} + \frac{-(1-\alpha_i)}{\alpha_i\left(I_b - I_e\frac{1-\alpha_i}{\alpha_i}\right)-I_{eo}}\right] \qquad (1\text{-}15)$$

APPENDIX V

This appendix derives an expression for α_n, the normal d-c current gain in terms of the transistor geometry and the transistor diffusion parameters. This expression is limited to p-n-p transistors possessing the geometry of the model used in Appendix I.

$$\alpha_n = -\frac{a_{21}}{a_{11}} \tag{A2-9}$$

a_{11} is found from equation (A1-27) to be,

$$a_{11} = \frac{q D_p p_n}{L_p}\left[\coth\frac{W}{L_p} + \frac{D_n n_p L_p}{D_p p_n L_n}\right] \tag{A5-1}$$

and a_{21} is found from equation (A1-29) to be,

$$a_{21} = \frac{q D_p p_n}{L_p}\operatorname{csch}\frac{W}{L_p} \tag{A5-2}$$

Equations (A5-1) and (A5-2) are substituted into (A2-9) to get

$$\alpha_{normal} = \frac{\dfrac{q D_p p_n}{L_p}\operatorname{csch}\dfrac{W}{L_p}}{\dfrac{q D_p p_n}{L_p}\left[\coth\dfrac{W}{L_p} + \dfrac{D_n n_p L_p}{D_p p_n L_n}\right]}$$

which can be simplified to,

$$\alpha_{normal} = \frac{1}{\cosh\dfrac{W}{L_p} + \dfrac{D_n n_p L_p}{D_p p_n L_n}\sinh\dfrac{W}{L_p}} \tag{1-18}$$

This appendix shows the details of the calculations which were made in obtaining curves V and VI of Fig. (16). The calculated values of V_{ec}, curves III and IV, fail to agree with measured values of V_{ec}, curves I and II. This disagreement is reconciled as follows:

The resistance of leads to the emitter and collector junctions is shown as R_{es} and R_{cs} in Fig. (21) which is duplicated below,

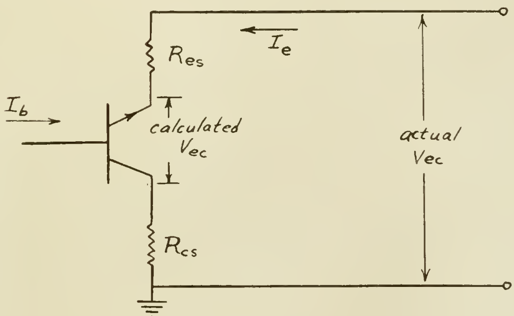

Figure (21)

The curves showing calculated values of V_{ec} do not include voltage drops caused by current flowing in these lead resistances. These voltage drops are included in the equation,

$$\text{Corrected } V_{ec} = I_e\,(R_{es} + R_{cs}) + I_b\,R_{cs} + \text{calculated } V_{ec}. \quad \text{(A6-1)}$$

This equation is tabluated below and solved for several points. Values of corrected V_{ec} are plotted as curves V and VI in Fig. (16).

I_e	$I_e(R_{cs}+R_{es}) = 1.4 I_e$	I_b	$I_b R_{cs} = 0.7 I_b$	Calc. V_{ec}	Corr. V_{ec}
0 ma	0 mv	1 ma	0.7 mv	0.53 mv	1.23 mv
1	1.4	1	0.7	1.61	3.6
2	2.8	1	0.7	2.64	6.14
-1	-1.4	1	0.7	-.8	-1.5
-2	-2.8	1	0.7	-2.1	-4.2
0	0	6	4.2	0.53	4.7
1	1.4	6	4.2	0.576	6.2
2	2.8	6	4.2	0.7	7.7
-1	-1.4	6	4.2	0.4	3.2
-2	-2.8	6	4.2	-0.5	1.3
-3	-4.2	6	4.2	-0.24	-0.24

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